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Modeling of Critical Threshold of Preference for Collusion (C.T.P.C.) and Cost Structure in Tunisian Mobile Market

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Abstract

We present in this paper a modeling of the critical threshold of preference for collusion (C.T.P.C) in different market structure using the interconnection fees and their marginal cost, in a Cournot competition. The objective is to compare the preference for collusion regarding this threshold in market structures and within two contexts: linear interconnection costs and quadratic ones. Collusion is more difficult in a private duopoly that in a mixed one. This difficulty is increased with linear cost structure than quadratic costs. The findings we obtain from the application of our results to the Tunisian mobile market between (2002-2019) are consistent with our theoretical model.

Keywords: Collusion, Costs Structure, Mixed Market.

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1. Introduction

The analysis of competition in the telecommunications market is generally used for modeling markets where competitors all persist in the same goal, namely maximizing their profits. However, there are markets where one or more firms pursue different objectives. This type of structure is called a mixed market. The new economy of the developed countries is a mixed economy in which the production of goods and services is carried out simultaneously by private firms and public firms (Ohnishi, 2011) .The public firm seeks to maximize the collective surplus given by the sum of the consumer surplus and the profit of the operator, while the private firm maximizes only its own profit.

The privatization movement that most countries have experienced in recent decades consists in converting a national company into a company owned and controlled by private investors. In the telecommunications sector, for example, the first countries that have privatized their public telecommunications operator are: Chile, Japan and the United Kingdom. Several studies have looked at this sector in France (Pénard, 2003), the United States (Madden & Savage, 2000, Parsons, 2002). In this sector, the year 2000 marked a significant turning point: more than half of the countries privatized all or part of their public operators. The economic stakes in the telecommunications sector and its regulation are very important. Several economic problems have to be solved, like the barriers to entry, networks interconnection (Colombier, M'Chirgui, & Pénard, 2010), privatization (Wallsten, 2002) and its effects on the nature (private or mixed) (De Donder, 2005) of markets, on their structure duopoly (Parker & Roller, 1997), Oligopoly (De Donder, 2005), and on the strategic behavior of competitors (competition, collusion) (Parker & Roller, 1997; Hoffler, 2009; Pénard, 2003; Souam & Pénard, 2002), agreement (Artz, Heywood, & McGinty, 2009) or deviation (Colombo, 2013), and Delbono and Lambertini (2014) have shown through a simple model of differentiated oligopoly that the nationalization of private firms can discourage tacit collusive behavior. In Tunisia, precisely in 2006, the privatization of "Tunisie Telecom" has transformed, from a theoretical point of view, the market structure from a mixed duopoly into a semi-private duopoly. In this context, an interesting question deserves to be asked: is a mixed duopoly preferable to a private duopoly in terms of collective welfare? Friedman (1971) Show that the answer to this question is not affirmative, even in the case where the public and the private firms have the same cost structure. In the same

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context, Tennbakk (1992) analyzed the profitability of mergers in an asymmetric mixed oligopoly composed of two private firms and a single public one with a quadratic cost structure, assuming that there is a technological gap between the two types of firms. They showed that the partial acquisition of a public firm with technological backwardness by a private firm can be beneficial in terms of collective welfare, but also for shareholders.

Moreover, Cortade (2005) studied the collusion strategy of internet operators and showed that the preference for collusion does not depend on the level of the access fees in the case where the market structure is vertically separated. We propose in this paper to model and compare the critical threshold of collusion in a market composed of two symmetric operators: the first one is public and the other is private, in a competitive Cournot context (Tennbakk, 1992). Our work will examine two cases: first, the interconnection costs are supposed to be linear, and in the second case these costs are supposed to be quadratic. We assume first a linear interconnection cost function that is common to both types of operators. The total cost of interconnection corresponds to the product of exchanged quantity and a parameter relating to the technology used and assumed to be identical for the two operators, which means that the marginal cost is constant. In a second case, we use a quadratic cost function with a linear interconnection cost that increases with the exchanged quantity.

The paper is organized as follows: first, we will present in section 2 the assumptions and the theoretical framework of the model. Then, in section 3, we introduce into the model the (C.T.P.C) in the context of a duopoly with mixed operators, using successively the two cases of linear and quadratic cost structures. In Section 4, we reintroduce the (C.T.P.C) in a duopoly where operators are no longer mixed, but private. Section 5 will focus on the results and a comparative analysis of the findings in the two previous sections. Finally, before concluding (section 7), we present in section 6 an application to the case of the Tunisian mobile telephony market which has passed through these structures.

2. Assumptions and General Framework

We assume that the mobile market is made up of two operators that are in a Cournot competition. Each operator is characterized by an interconnection cost θ_i (Flochel, 1999; Hoffler, 2009). There are two types of traffic on telecommunication markets: an "on-net" internal traffic between consumers connected to the same network, and an "off-net" external traffic between consumers connected to different networks. In this second type of traffic, the networks then tariff each other an access charge in order to be able to carry a communication using the competitor network (Hoernig, 2007). Each operator i can support two types of costs: the cost of getting his call to the point of interconnection with his competitor j (this cost depends on the amount of outgoing calls q_{ij}) and the cost of routing a call from Competitor on its own network (this cost depends on the number of incoming calls q_{ji}). We take into account only the exchanged quantities between the operators on the off-net traffic (q_{ij}, q_{ji}), seeing that the monetary transfers (the most incentive factor for collusion[†] concerns only the external traffic and not internal one (q_{ii}, q_{jj}). Assume that q_{12} is the quantity of traffic flowed from network 1 to network 2 and q_{21} the quantity of traffic flowed from network 2 to network 1. The two operators agree on a common interconnection tariff $a_1 = a_2 = a$. More specifically, the amount a corresponds to the amount that each operator can receive from its competitor for a one minute use of his own network. We also assume that the two operators practice very similar retail prices $P_1 = P_2 = P$ (this is the case for mobile operators in Tunisia). Let Q denote the total quantity of traffic exchanged between the two networks and $P = 1 - Q = 1 - (q_{12} + q_{21})$ the inverse demand function. We model the critical threshold of collusion in two different situations: the case where the duopoly is mixed and the one where it is private. In each of these two industrial configurations, we assume successively that the costs are first linear and then quadratic. The first operator *(i=1)* is assumed to be the private operator. Its profit function, when the cost structure is supposed to be linear, is:

$$\pi_1 = (1 - q_{12} - q_{21} - \mathrm{a})\, q_{12} + m q_{21} \qquad \text{Where } m = a - \theta \textit{ is the access margin}$$

The second Operator is assumed to be the public actor. While the private firm maximizes its profit π_1, operator 2 maximizes the collective surplus π_2 given by the sum of consumers' surplus and its own profit.

[†]The most incentive factors for collusion are presented in Pénard, T. (2002)

$$\pi_2 = \frac{1}{2}(q_{12}+q_{21})^2 + (1-q_{12}-q_{21}-a)q_{21} + mq_{12}$$

Finally, we assume the hypothesis of Debbichi, S. and al. according to which both operators use the same technology and charge an interconnection tariff that is higher than the marginal cost. The incitation to collude in the mobile market will depend on the values of the (C.T.P.C) and the discount factor. In fact, the choice between the two strategic behaviors (competitive or collusive) of the two operators is based on the trade-off between short-term gain to deviate from collusion and long-term loss when there is deviation. After a deviation, there will be a return to a competitive situation in the following period. Collusion for an operator is sustainable if the preference for the present measured by δ and expressed by the discount rate r, where $\delta = \frac{1}{1+r}$ and $0 \leq \delta \leq 1$, is sufficiently low [17]. The sequential game is as follows: assume that an agreement is reached between the two operators in t = 0. When this agreement is respected, the operator i realizes a collusion profit π_i^{Col}. If the operator i decides to deviate from the cartel, he obtains a deviation profit $\pi_i^{Dev} > \pi_i^{Col}$. The short-term gain is then equal to $\pi_i^{Dev} - \pi_i^{Col}$. In the long term, the present value of gains in case of deviation is:

$$\mathrm{V}_{\mathrm{i}}^{\mathrm{Co}} = \sum_{\mathrm{t}=1}^{\infty} \delta^{\mathrm{t}}\, \pi_{\mathrm{i}}^{\mathrm{Comp}} = \frac{\delta}{1-\delta}\pi_{\mathrm{i}}^{\mathrm{Comp}}$$

And the present value of collusion is:

$$\mathrm{V}_{\mathrm{i}}^{\mathrm{Col}} = \sum_{\mathrm{t}=1}^{\infty} \delta^{\mathrm{t}}\, \pi_{\mathrm{i}}^{\mathrm{Col}} = \frac{\delta}{1-\delta}\pi_{\mathrm{i}}^{\mathrm{Col}}$$

In the long run, as the profit from competition is lower than the benefit of collusion ($\pi^{Col} > \pi^{Comp}$), the discounted loss suffered by an operator following the decision to deviate is equal to:

$$\frac{\delta}{1-\delta}(\pi^{Col} - \pi^{Comp})$$

The operator will not be strategically interested in deviating when the long-term discounted loss is greater than the short-term gain obtained from deviation. In other words, when:

$$\frac{\delta}{1-\delta}\left(\pi^{\mathrm{Col}} - \pi^{\mathrm{Comp}}\right) > \left(\pi_{\mathrm{i}}^{\mathrm{Dev}} - \pi_{\mathrm{i}}^{\mathrm{Col}}\right) \Leftrightarrow \delta > \bar{\delta} = \frac{\pi^{\mathrm{Dev}} - \pi^{\mathrm{Col}}}{\pi^{\mathrm{Dev}} - \pi^{\mathrm{Comp}}}$$

Where $\bar{\delta}$ is defined as the critical threshold of preference for collusion.

3. Duopoly with mixed operators‡

3.1. Linear Interconnection Costs

The operators in the mobile telephony market bear two types of costs: the interconnection tariff charged by the operators and the total interconnection cost paid by each firm depending on the technology used. There are two types of interconnection costs: the first one relates to direct fixed interconnection costs, which are costs related to the installation of new specific facilities to ensure interconnection. This can be simple extensions of the network, or more complex installations requiring large investments. The second type of costs is the indirect variable costs of interconnection, which are sensitive to traffic. The total linear interconnection cost $CT_i = \mu_i q_{ji}$ depends on the technologyμ_i, assumed to be identical ($\mu_i = \mu_j = \mu$) for the two operators, and the quantity q_{ij} exchanged between the two networks, with $i,j = \{1,2\}\ and\ i \neq j$.The marginal cost of interconnection given by $\theta_i = \frac{\partial CT_i}{\partial q_{ij}} = \mu_i = \mu$§ is constant and therefore does not depend on the exchanged quantity. The marginal cost is the additional cost, added to an existing cost base, necessary to ensure a specific marginal development for a given service.

3.1.1. The Cournot Competition

The two operators use an interconnection tariff that is lower than the marginal cost, which ensures them a gain equal

‡ The quantities and profits formula are in table 1 Appendix1.

§One can follow the following proposition: μ = [0,1], where μ = 1 corresponds to the best technology adopted and μ = 0 Corresponds to the oldest technology.

to $a - \theta = a - \mu$. Taking account of this gain, we obtain the following profit for each operator: While the public operator (operator 2) maximizes the collective surplus given by the sum of the consumer surplus and of his own profit, the private operator maximizes his own profit. The first-order conditions of the two maximization programs are obtained with $\frac{\partial \pi_i^{Comp}}{\partial q_{ij}} = 0$, where $i, j = \{1,2\}\ and\ i \neq j$:

$$\pi_1^* = (a - \mu)(1 - a)\ and\ \pi_2^* = \frac{1}{9}(1 - a)(1 + 2a - 3\mu)$$

According to these results, the profit of the public operator depends only on the level of the interconnection tariff practiced, whereas the profit of the private operator depends on both, the technology and the level of the interconnection tariff. The private operator takes advantage of the incoming traffic q_{21} only, whereas the public operator takes advantage of only the outgoing traffic q_{21}. Let S_1 and S_2 be the market share respectively of operator 1 (the private operator) and operator 2 (the public operator), where $S_1 + S_2 = 1$. It follows that:

$$S_1 = \frac{q_{12}^*}{Q^*} = 0 \qquad et \qquad S_2 = \frac{q_{21}^*}{Q^*} = 1$$

Proposition 1: *Under a Cournot competition regime and a linear cost structure, the private operator has no weight on the market and the public operator is the only supplier. The profit of the public operator depends only on the interconnection tariff, while the profit of the private operator depends also on the used technology.*

3.1.2. Equilibrium in case of collusion

The gains derived by the two operators from a potential collusion[**] decision are calculated by maximizing the joint profit noted π_M. The two firms behave in this case like a monopoly. The two firms arrange to share the profit of the monopoly according to a rule fixed by mutual agreement. The joint profit function is written as follows:

$$\pi^{Col} = \frac{1}{4}(1 - \mu)^2$$

Where Q_M represents the total quantity of interconnection produced by the two firms. Finally, the equilibrium quantity resulting from this cooperation is given by:

$$Q_M^* = (1 - \mu) \text{ et } \pi_M^* = \frac{1}{2}(1 - \mu)^2$$

In real markets, we observe that the quantities produced in case of collusion between firms, following the application of a cooperation strategy, are allocated in proportion to the market shares. We will assume for simplification concerns that the two operators share equally the total profit made in case of collusion. The profit obtained in presence of collusion depends only on the technology used, and is independent of the level of interconnection tariff, which allows us to announce the following proposition:

Proposition 2: *In a Cournot competition regime where the two operators agree to cooperate, and with a linear interconnection cost structure, the quantities and profits of the two (private and public) operators vary according to the used technology and do not depend on the interconnection tariff.*

3.1.3. Equilibrium in case of deviation

To calculate the profit in case of deviation, we assume that the leading (private) operator chooses to deviate, while assuming that its rival (the public operator) maintains constant its output level resulting from the strategy of collusion. The private operator chooses then the quantity of deviation denoted q_{12}^{dev} that maximizes its profit function.

$$\pi_1^{\mathrm{Dev}} = \frac{1}{16}(1 + \mu - 2a)^2 + \frac{1}{2}(1 - \mu)(a - \mu)$$

Since the quantity produced by operator 2 is unchanged and equal to $q_{21}^{col} = \frac{1}{2}(1 - \mu)$, we can therefore announce the following proposition:

[**] The maximization of the joint profit is a convenient but reductive compromise in many ways. This compromise is all the less justified here as firms are structurally asymmetric: one is private and the other one is public. Their incentives to share the joint profit in the cartel are structurally different.

Proposition 3 : *In a duopolistic competition and with a linear cost structure, if the private operator deviates[††], the two operators have non-zero market shares that vary according to the used technology (for both operators) and to the value of the interconnection tariff (for the private operator).*

3.1.4. Calculation of the (C.T.P.C)

After calculating successively the different profits obtained in cases of competition, collusion and deviation, in presence of a linear cost structure, we can calculate the (C.T.P.C) from the formula announced previously: We find then:

$$\delta > \bar{\delta} = \frac{\frac{1}{16}(1+\mu-2a)^2+\frac{1}{2}(1-\mu)(a-\mu)-\frac{1}{4}(1-\mu)^2}{\frac{1}{16}(1+\mu-2a)^2+\frac{1}{2}(1-\mu)(a-\mu)-(a-\mu)(1-a)}$$

when $\mu = 1$ and $\forall$ a $\in [0,1; 1]$, the(C.T.P.C) is equal to $\delta > \bar{\delta} = \frac{1}{5} =$ constant.

3.2. Quadratic Interconnection Costs

We repeat the same procedure as before in the previous subsection, keeping the same assumptions except for the cost structure that is assumed henceforth to be quadratic. Suppose that he quadratic total interconnection cost is $CT_i = \mu_i q_{ji}^2$. This cost depends on the technologyμ_i, that is always assumed to be identical for the two operators ($\mu_i = \mu_j = \mu$), and the quantity q_{ij} exchanged between the two networks, where$i, j = \{1,2\}$ and $i \neq j$. It should be noted that for $i, j = \{1,2\}$, the marginal cost of an interconnection unit of a network i does not depend on the outgoing quantity of network i to network j, but on the incoming quantity of network j to network i, $where$ $i \neq j$. The marginal cost of interconnection of operator i is given by $\theta_i = \frac{\partial CT_i}{\partial q_{ij}} = 2\mu_i q_{ji}$. In other words, for a quantity q_{ji} equal to unity (ie for one minute of traffic), the technology used is such that$\mu_i = \frac{1}{2}\theta_i$. In this case, the marginal cost of interconnection is linear and depends not only on the technology used, but also on the quantity exchanged between the two networks, with a slope equal to $2\mu_i = 2\mu$.

3.2.1. The Cournot Equilibrium

The two operators are supposed to practice a price that is lower than the marginal cost, which gives them an access margin equal to $a - \theta = (a - 2\mu q_{ji})$. Taking account of this access margin, the profits and the exchanged quantities of operators represented in table 1 .The maximization of the profit of each operator ($\frac{\partial \pi_i^{Comp}}{\partial q_{ij}} = 0$, where $i, j = \{1,2\}$ and $i \neq j$) gives in table1. Cournot's equilibrium profits is equal to: $\pi_1^* = \big(a - 2\mu(1-a)\big)(1-a)$

According to these results, as in the case with linear interconnection costs, the profit of the public operator depends only on the level of interconnection tariff, while the profit of the private operator depends both on the technology and the level of interconnection tariff. The market shares ($S_1 = 0$ and $S_2 = 1$) are also the same as when the costs were linear. The only difference concerns the profit of the operator 1 which changes when the interconnection costs become quadratic. The situation of a monopoly with only one public operator can be explained by the fact that this operator is the incumbent holding and controlling the infrastructures, and can consequently prevent the private operator from accessing its network ($q_{12}^* = 0$). The public operator is therefore the only service provider. We announce then the following proposition:

Proposition 4: *In a Cournot competition regime and assuming a quadratic costs structure (linear marginal costs), the private operator has no weight on the market and the public operator is the only supplier. The profit of the public operator depends only on the interconnection tariff, while the profit of the private operator depends also on the used technology.*

At the Cournot equilibrium, comparing the profit of firm 1 (the private operator) when the interconnection costs are quadratic, with its profit obtained in the previous case where these costs were linear, we find that, if $\mu(2a - 1) > 0$,

[††] If we suppose that the public operator chooses to deviate and that the private operator maintains constant its output level resulting from the collusion strategy, the deviation quantity denoted q_{21}^{dev} that maximizes the profit function of the public operator is: $\frac{1}{8}(3 - \mu - 2a)^2 + \left(\mu - \frac{1}{2}\right)(1-\mu)(a-\mu)$

firm 1 realizes a greater profit when the cost structure is quadratic. For positive values of μ, this condition becomes $a > 1/2$. However, for the profit of firm 2 (the public one), it remains unchanged at Cournot's equilibrium, when the cost structure changes.

3.2.2. Equilibrium in case of Collusion

In order to estimate the gains realized following a decision of potential collusion between the two operators, we maximize the joint profit, noted π_M and shared by the two firms according to an arrangement between them , Baranes, E., and al.,(2012) . Q_M represents the total quantity of interconnection produced by the two operators. As for the total profit resulting from the strategy of cooperation between the two operators'is equal to:

$$\pi_M^* = \frac{1}{2(1+4\mu)}$$

As the latter expression shows, the benefit of collusion depends only on the used technology and seems to be independent of the level of interconnection tariff. In real markets, the collusive quantities resulting from the cooperation strategy are allocated in proportion to the market shares. We assume for simplicity that the two operators share equally the profit of collusion. Each one of them thus obtains a profit π^{Col} such that: $\pi^{Col} = \frac{1}{2}\pi_M^* = \frac{1}{4(1+4\mu)}$

Proposition 5: *In a Cournot competition regime where the two operators agree to cooperate, and with a quadratic interconnection costs structure, the quantities and profits of the two (private and mixed) operators vary with the used technology and are Independent of the interconnection tariff.*

By comparing the profit of firm 1 (the private firm) in the case of collusion, when interconnection costs are quadratic, with its profit obtained in the previous case where these costs were linear, we find that if $(1-\mu)^2.(1+4\mu) < 1$, firm 1 realizes a greater profit when the cost structure is quadratic.

3.2.3. Equilibrium in Case of Deviation

To calculate the profit in the event of deviation, we assume that it's the leading operator (the private one)[‡‡] that chooses to deviate. In such a case, its rival continues to keep his output level constant and equal to the output q_{21}^{col} calculated in the collusion case. The private operator chooses the deviation quantity, notedq_{12}^{dev}, that maximizes his own profit π_1^{Dev} equal to:

$$\pi_1^{\mathrm{Dev}} = \left(\frac{1+8\mu}{4+16\mu} - \frac{1}{2}\mathrm{a}\right)^2 + \frac{1}{2}\left(\mathrm{a} - \frac{\mu}{(1+4\mu)}\right).\frac{1}{(1+4\mu)}$$

Proposition 6 : *In a duopolistic competition with a quadratic cost structure, when the private operator deviates, the two operators continue having non-zero market shares that vary according to the used technology (for both operators) and to the value of the interconnection tariff (for the private operator).*

By comparing, in a mixed duopoly, the profit of firm 1 (the private one) in case of deviation, when the interconnection costs are quadratic, with its profits obtained in the previous case where these costs were linear, we find that this profit may vary depending on the values of "a" and μ. In terms of market shares, firm 1 gains more shares when the costs are quadratic in comparison to its shares when these costs are linear, if the used technology used is such that $\mu < 3/4$.

3.2.4. Calculation of the critical threshold of collusion

After calculating successively the different profits obtained in competition, collusion and in case of deviation, when the cost structure is quadratic and when the market is mixed, we can calculate the (C.T.P.C) using the formula calculated previously and for $\mu = 1$ and $\forall$ a $\in [0,1;1]$, a(C.T.P.C) equal to:

[‡‡]If we are interested in the case where the deviation concerns the public operator, $\frac{1}{8}(3-\mu-2\mathrm{a})^2 + \frac{1}{2}(\mu-1)(1-\mathrm{a}) + \frac{1}{2}(1-\mu)\big(\mathrm{a}-\mu(1-\mu)\big)$

$$\delta_1 > \bar{\delta} = \frac{\left(\frac{9}{20}-\frac{1}{2}\mathrm{a}\right)^2+\left(\frac{1}{10}\mathrm{a}-\frac{1}{50}\right)-\frac{1}{20}}{\left(\frac{9}{20}-\frac{1}{2}\mathrm{a}\right)^2+\left(\frac{1}{10}\mathrm{a}-\frac{1}{50}\right)-(3\mathrm{a}-2)(1-\mathrm{a})}$$

So what happens in the situation where the duopoly is purely private (with two private operators)?

4. A Duopoly with private operators

In this section, we will compare the two (linear and quadratic) cost structures, when assuming the presence of two private operators. The order presented in our study corresponds to the chronology of the changes observed in the market. In fact, in recent years we have witnessed a wave of privatization of incumbent operators, as well as the expansion of the number of players in the telecommunications industry. We focus here on the privatization process, and we are not concerned about the effect of the change in the number of actors.

4.1. Linear Interconnection Costs

We assume that the interconnection cost between the two operators is different from zero and that they charge each other a tariff above this interconnection cost. Let $CT = \mu_i q_{ji}$ be the linear total interconnection cost. It depends on the used technology μ_i, still assumed to be identical for the two operators, and on the quantity q_{ij} exchanged between the two networks, with $i, j = \{1,2\}$ and $i \neq j$.

4.1.1. The Cournot Equilibrium

The first order conditions and the resolution of the profit maximization program of the two firms give us the following function:

$$\pi_1^* = \pi_2^* = \frac{(1-a)}{9}(1+2a-3\mu)$$

When the cost structure is linear, in order to have a greater profit in situation of private duopoly than in situation of a Cournot mixed duopoly, the value of μ should be greater than $\frac{(7a-1)}{6}$.Contrary to what is announced in proposal 1, the weight of each operator in this duopoly framework is different from zero and the market share of each operator depends on the interconnection tariff. The private operator (operator 1) increases its market share (for $a > 0$) when operator 2 becomes private. The latter, on the other hand, sees its market share declining as a result of this change. However, when the duopoly becomes private, the used technology does not affect the market shares of the two firms, but has an effect on the profit of the two operators.

4.1.2. Equilibrium in case of collusion

In case of collusion, the two private operators maximize the joint profit noted π_M and agree on the joint profit and where Q_M is the total quantity of interconnection produced by the two operators that behave like a in a monopoly. The quantities and profits in case of collusion when the interconnection costs are linear and when the two operators are private, are halved compared to the case where one of the two operators is public and equal to:

$$\pi^{Col} = \frac{1}{8}(1-\mu)^2$$

However, as in proposal 2, the quantities and profits of the two private operators vary with the used technology and are independent of the interconnection tariff.

4.1.3. Equilibrium in case of deviation

We assume for instance that it's operator 1 that deviates from the collusion agreement and that operator 2 continues to produce the quantities decided in the collusive framework. The maximization of this profit gives an optimal quantity to be *produced* by operator 1 in case of deviation equal to $\mathrm{q}_{12}^{\mathrm{dev}} = \frac{1}{8}(3+\mu-4\mathrm{a})$. In presence of linear interconnection costs, this quantity is greater than the quantity obtained following a deviation, in the case where the second operator was public, if $\mu < 1$. Such a quantity produced by the operator who chooses to deviate from the collusive framework allows the realization of a profit equal to:

$$\pi_1^{\mathrm{Dev}} = \left(\frac{3}{8} + \frac{1}{8}\mu - \frac{1}{2}a\right)^2 + \frac{1}{4}(a-\mu)(1-\mu)$$

The above-mentioned *proposition 2* therefore remains valid in the case where interconnection costs are linear and when the two operators are private, since the market shares of the two firms are different from zero and vary according to the used technology (for The two *operators*) and to the value of the interconnection tariff (for the deviating private operator).When the cost structure is linear, the profit made by firm 1 in the case of deviation and in presence of two private firms is greater than its profit obtained when deviating in a mixed duopoly market, if $a < \frac{5+14\mu-19\mu^2}{8\mu+24}$.

4.1.4. Calculation of the (C.T.P.C)

After calculating successively the different profits obtained in competition, collusion and deviation, when the cost structure is linear and when the two operators are private, we can calculate the (C.T.P.C) using the formula proposed previously, we obtain:

$$\bar{\delta} = \frac{\left(\frac{3}{8} + \frac{1}{8}\mu - \frac{1}{2}a\right)^2 + \frac{1}{4}(a-\mu)(1-\mu) - \frac{1}{8}(1-\mu)^2}{\left(\frac{3}{8} + \frac{1}{8}\mu - \frac{1}{2}a\right)^2 + \frac{1}{4}(a-\mu)(1-\mu) - \frac{1}{9}(1-a)(1+2a-3\mu)}$$

For a value of $\mu = 1$ and for values of $a \in [0,1; 1]$, we obtain a (C.T.P.C) equal to $\bar{\delta} = \frac{9}{17} =$ Constant.

4.2. Quadratic Interconnection Costs

Let $CT_i = \mu_i q_{ji}^2$ be the quadratic total cost of interconnection. It depends on the technology μ_i that is still assumed to be identical for the two operators and the quantity q_{ij} exchanged networks $i, j = \{1,2\}$ where $i \neq j$. . It should be noted that for $i, j = \{1,2\}$, the marginal cost of an interconnection unit of a network i does not depend on the outgoing quantity of network i to network j, but on the incoming quantity of network j to network i, $where\ i \neq j$. The marginal cost of interconnection of operator i is given by $\theta_i = \frac{\partial CT_i}{\partial q_{ij}} = 2\mu_i q_{ji}$. It depends not only on the used technology, but also on the quantity exchanged between the two networks. For a quantity equal to unity, or say for one minute of traffic, we have$\mu = \frac{1}{2}\theta$. We assume that the interconnection cost is different from zero and that the two operators charge each other a tariff that is higher than this cost.

4.2.1. The Cournot Equilibrium

With quadratic interconnection costs and two private operators, the reaction functions from the profit maximization program are $q_{12}(q_{21}) = \frac{1}{2}(1 - q_{21} - a)$ and$q_{21}(q_{12}) = \frac{1}{2}(1 - q_{12} - a)$. At equilibrium, the two firms exchange the same quantities on the market$q_{12}^* = q_{21}^* = \frac{1}{3}(1-a)$. This is the same quantity as that in the case where the cost structure was linear. When the duopoly becomes private and the cost structure becomes quadratic, the market shares of the two firms remain unchanged compared to the case where the cost structure was linear. These shares are different from zero, in contrast to the case where the duopoly is mixed. The two firms realize the same profit equal to:

$$\pi_1^* = \pi_2^* = \frac{1}{9}(1-a)(1 + 2a(1+\mu) - 2\mu)$$

When the duopoly is private, this profit, that depends on both the used technology and the interconnection tariff, is higher than the profit realized in the case where the cost structure is linear if $a < \frac{1}{2}$. When the cost structure is quadratic, in a Cournot situation, the profit made in the presence of two private operators is greater than the profit obtained by the private firm when the second firm is public, if a$< \frac{1+16\mu}{7+16\mu}$.

4.2.2. Equilibrium in case of collusion

The two operators maximize the joint profit noted π_M and agree on the sharing rule. The two operators behave like in a monopoly. Where Q_M is the total quantity of interconnection produced by the two firms. The quantity resulting from this cooperation at the equilibrium is $Q_M^* = \frac{1}{2(1+2\mu)}$, which corresponds to a same quantity for each firm equal to

$q_{12}^{col} = q_{21}^{col} = \frac{1}{4(1+2\mu)}$. The profit realized is, which corresponds to a production share for each firm equal to: $\pi^{Col} = \frac{1}{8(1+2\mu)}$

In such a framework where the duopoly structure is private and the costs are quadratic, these quantities produced in case of collusion by firm 1 are higher than those produced in the same framework by this firm when the costs are linear, if $\mu(1-2\mu) < 0$, which corresponds to the condition $\mu > \frac{1}{2}$ if we assume logically that μ takes only positive values. Moreover, when the costs become quadratic, proposition 2 remains valid in case of collusion. In fact, the produced quantities, as well as the profits, depend only on the used technology.

4.2.3. Equilibrium in case of deviation

We assume that it's operator 1 that deviates from the collusion agreement and that operator 2 continues to produce the quantities decided in the collusive framework. Under this assumption, the profit of operator 1 in case of deviation corresponds a reaction function equal to $q_{12}^{dev} = \frac{1}{2}(1 - q_{21}^{col} - a)$ and a quantity produced by firm 1 equal to$q_{12}^{dev} = \frac{3+8\mu(1-a)-4a}{8(1+2\mu)}$. We continue to assume that firm 2 respects the collusion agreement and therefore produces a quantity equal to $q_{21}^{col} = \frac{1}{4(1+2\mu)}$. The profit of firm 1 after deviation from collusion is equal to:

$$\pi_1^{\mathrm{Dev}} = \left(\frac{3 + 8\mu - 8\mu a - 4a}{8(1+2\mu)}\right)^2 + \left(\frac{4a + 8a\mu - 2\mu}{16(1+2\mu)^2}\right)$$

It follows that *proposition 3* and *proposition 6* remain valid when the cost structure becomes quadratic and when both operators are private. When the cost structure is quadratic and the two firms are private, the market share of the firm that deviates from collusion is higher compared to the case where the costs structure is linear, if the technology used is such that$\mu < \frac{1}{2}$. Moreover, a comparison of the two cases (mixed operators and private operators) where the cost structure is quadratic shows that the market share of the firm that deviates when the two firms are private is higher than its share when one of the firms is public if $\mu < \frac{1}{\sqrt{32}}$.

4.2.4. Calculation of the (C.T.P.C)

After calculating successively the different profits obtained in competition, in collusion and in case of a deviation, when the cost structure is quadratic and when the two operators are private, we can calculate the (C.T.P.C) from the formula proposed above. After replacing each term in the latter expression by its value calculated previously, and or$\mu = 1$ and $\forall$ a $\in [0{,}1; 1]$, we obtain a (C.T.P.C) equal to:

$$\bar{\delta} = \frac{\left(\frac{11-12a}{24}\right)^2 + \frac{1}{12}\left(a - \frac{1}{6}\right) - \frac{1}{24}}{\left(\frac{11-12a}{24}\right)^2 + \frac{1}{12}\left(a - \frac{1}{6}\right) - \frac{1}{9}(1-a)(4a-1)}$$

5. Oligopoly with three private operators

In this section we will try to reproduce the same approach as the previous sections, but in the case of three operators$i, j\ et\ k\ with\ \ i,j,k\ \in \{1,2,3\}\ avec\ i \neq j \neq k$. And at this time with linear costs After calculating the differents expressions of profits we get the following threshold formula :

$$\overline{\delta} = \frac{\frac{3}{4}a^2 - \frac{2}{3}a + 3/54}{\frac{3}{4}a^2 - \frac{2}{3}a + \frac{1}{9} + \frac{1}{16}(1-a).(1-3a)}$$

6. Results and Comments

After studying the different combinations of cost structures and market structures, the (C.T.P.C) $\bar{\delta}$ can be written as $\bar{\delta} = \frac{f(a,\mu)}{g(a,\mu)}$ where $f(a,\mu) = \pi^{Dev}(a,\mu) - \pi^{Col}(a,\mu)$ and $g(a,\mu) = \pi^{Dev}(a,\mu) - \pi^{*}(a,\mu)$. The first term $f(a,\mu)$ denotes the interest of deviating from the collusive equilibrium, while the second term $g(a,\mu)$ denotes the advantage of deviating from the competitive equilibrium. Consequently, if the value of $\bar{\delta}$ is close to zero, then $f(a, \mu) = \pi^{Dev}(a, \mu) - \pi^{Col}(a, \mu) = 0$, which means that the profit resulting from deviation is equal to the profit resulting from the strategy of collusion. This can then stabilize collusion between the two operators and makes any deviation uninteresting. Moreover, when the value of the discount factor δ, assumed to be between 0 and 1, is greater than the (C.T.P.C) $\bar{\delta}$, the operator should prefer collusion [12] F*igure 3* shows the variation of the (C.T.P.C) $\bar{\delta}$ as a function of the value of the interconnection tariff a $\in [0.1; 1]$ in the case of a mixed duopoly and when the used technology is such that $\mu = 1$. When the interconnection cost structure is linear, the (C.T.P.C) $\bar{\delta}$ is constant and equal to $\frac{1}{5}$. In this case, the low value of the threshold $\bar{\delta}$ makes collusion very probable.

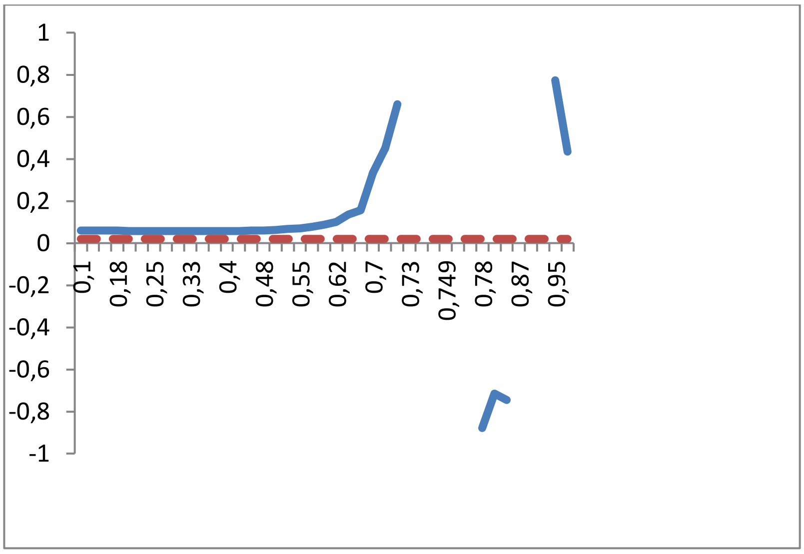

Figure 1. Variation of the (C.T.P.C) as a function of the interconnection tariff "a" in a mixed duopoly

When the interconnection cost structure is quadratic, the value of the (C.T.P.C) is positive for any value of "a" in the intervals [0.1; 0, 74] and [0, 87; 1]. The theoretical value of this threshold is lower than 1, value under which collusion becomes possible. Recall that the lower this value is, the easier collusion is. Therefore, the comparison of the case where the interconnection costs are linear with the case where these costs are quadratic shows that, for values of "a" in the interval [0;0.7], the threshold is lower in the quadratic case and thus the collusion is easier to set up. Contrary to the quadratic case, collusion therefore seems to be more difficult in the case where the interconnection costs structure is linear. This confirms, indeed, our intuition that, the more expensive the production is, the more easily the two companies agree on a tariff. Except for certain values of "a" in the intervals [0, 7; 0.74] and [0.99; 1], for which the value of the threshold is higher in the quadratic case, the following proposition holds for the rest of the values of "a" on the interval [0; 1]:

Proposition 7: *in a mixed duopoly and with a used technology such that* $(\mu = 1)$[§§]*, collusion is more difficult when the cost structure is linear than in the case where this structure is quadratic.*

In the case of a private duopoly (*Figure 2*), and when the interconnection cost structure is linear, the value of the (C.T.P.C) remains constant and does not vary with the value of the interconnection tariffs a $\in [0; 1]$. However, this threshold is higher $(\bar{\delta} = 0.53)$ than the previous case with mixed duopoly $(\bar{\delta} = 0.2)$. Collusion is therefore more difficult in a private duopoly than in a mixed duopoly, when the interconnection costs structure is linear. Still in the context of a private duopoly (*Figure 2*), when the cost structure is quadratic, the (C.T.P.C) is between zero and 1 for values of the interconnection tariff "a" belonging to [0 ; 0.4] and [0.81 ; 1]. When the value of "a" belongs to these intervals and when the value of the discount factor δ is greater than the value of the (C.T.P.C) $\bar{\delta}$ corresponding to the value of "a" in these intervals, collusion Becomes possible and preferred to deviation.

Proposition 6 therefore seems no longer valid when the duopoly becomes private, since in a private duopoly, and contrary to the mixed duopoly, collusion is easier when the cost structure is linear than when it is quadratic (except for

[§§] Simulation with $\mu = 1$ corresponds to the best technology used as this is reported in the text. One of the possible theoretical extensions of this work is to vary μ (the variation of μ corresponds to the variation of the technology level).

values of the interconnection cost "a" close to 0 or to 1). This result seems to be very intuitive, because for market actors, the more expensive the production is, the more easily the companies agree on a tariff. By comparing the two cases where the cost structures are quadratic, we observe that collusion is easier when the duopoly is private (a∈ [0; 0.4] and [0.81; 1]) than when the duopoly is mixed (a ∈ [0.1, 0.74] and [0.87; 1]). We can then announce the following proposition:

Proposition 8: in a private duopoly and with a used technology such that μ = 1, collusion is more difficult when the cost structure is linear and easier when the cost structure is quadratic, than in the case of a mixed duopoly.

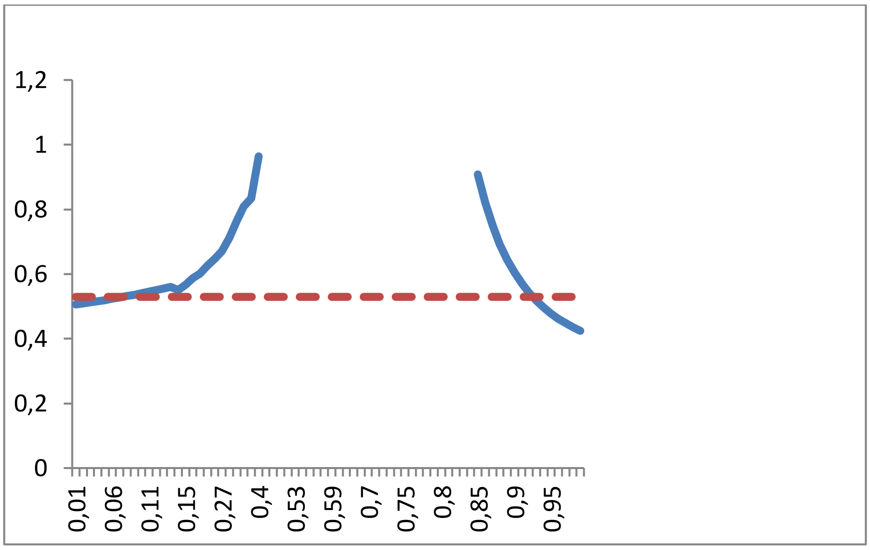

Figure 2. Variation of $\bar{\delta}$ as a function of the interconnection tariff "a" in a private duopoly.

7. Application to the case of the Tunisian mobile telephony market

In Tunisia, mobile operators operate under the CPNP regime. That means the caller's network pays the operator who receives the call Gruber, H. (2005). This is the most used rule in the mobile telephony markets. The monetary transfer between the operators is an important factor that facilitates collusion. [9]"... Is the fact that there has been little decrease in the evolution of interconnection tariffs after 2008, despite some stability in the duopoly period between 2003 and 2008" The market structure in Tunisia has undergone several changes, starting with a monopolistic structure (1992-2001), to a duopoly (2002-2009).The privatization of "Tunisie Telecom" in 2006 and the entry of "Orange Tunisia" on the market in 2010, the market became composed (theoretically) of three private operators. By applying our theoretical results announced previously for the calculation of the (C.T.P.C) to the evolution of the mobile phone market in Tunisia between 2002 and 2019, we obtain the following figure3. We will use these different real values of "a» to compare the different values of the theoretical expressions of the (C.T.P.C) calculated with$\mu = 1$. Debbichi, S. and al.,. (2013) also estimated the interconnection costs in Tunisia over the same period 2002-2010, when the cost structure is linear. We will use these costs and complete the estimation values to 2019 for purposes of comparison with the different calculated values of the thresholds of collusion and with the interconnection tariff "a".

When the used technology allows a linear cost structure, the (C.T.P.C) $\bar{\delta}$ do not depend of the interconnection tariff "a" and therefore remains constant. The value of this threshold for both duopoly structures (private and mixed) is higher than the estimated interconnection cost and the interconnection tariff "a". According to the results presented in *figure3*, the value of $\bar{\delta}$ (C.T.P.C) is higher when the duopoly is private than in the case where the duopoly is mixed with linear cost. With three private operators the value of (C.T.P.C) decreases. In fact, in 2006, with the privatization of the public operator "Tunisie Telecom", competition on the market became more severe and collusion became more difficult to achieve. This result is in contradiction with the conclusions of Delbono and al.(2014) that show that the nationalization of private firms can discourage tacit collusive behavior. But with the entrance of a third private operator collusion became easy to achieve.

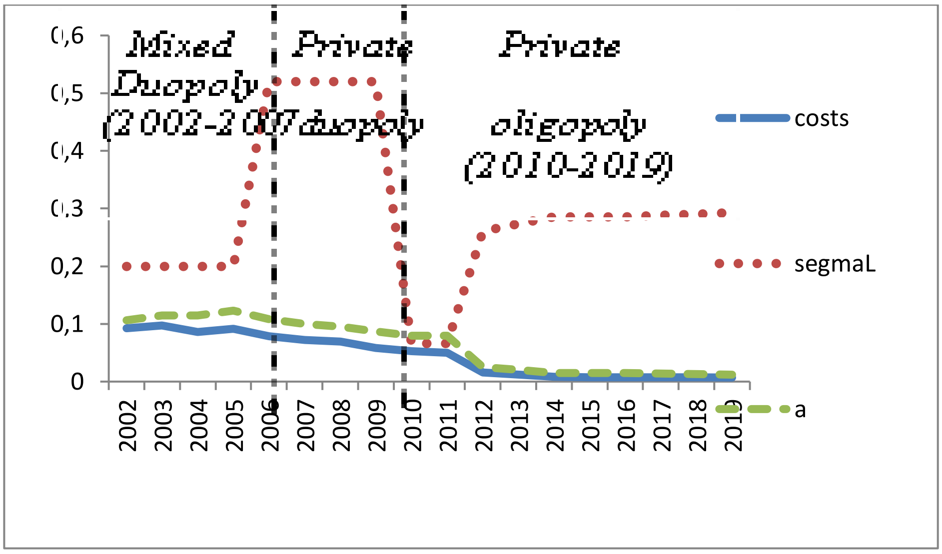

Figure 3. (C.T.P.C) estimated interconnection cost and real interconnection tariff on the Tunisian mobile market.

8. Conclusion

In this paper, the (C.T.P.C) depend, of a tariff “a” and a parameter μ relating to the technology used by the operator. We can retain a recapitulative result of our work: Collusion is more difficult in a private duopoly than in a mixed duopoly. This difficulty is more acute with a linear cost structure than with a quadratic cost structure. In the case of a mixed duopoly, with linear costs, the threshold is higher when the public operator chooses to deviate than when the private operator deviates. The threshold of preference for collusion is a relevant indicator available to the regulator to estimate operators' preferences for collusion. In order to establish a competitive system, the regulator can set the interconnection tariff and the marginal cost at a level that minimizes the preference for collusion. The case of the Tunisian market, taken as an example of markets in developing countries, allows us to calculate the different theoretical values of the (C.T.P.C), as well as its variation in a dynamic sector that has changed its structure from a mixed duopoly to a private duopoly. The regulator can use our results to better control the state of competition in the market by acting on the state of technology, the interconnection tariff, and the (C.T.P.C) One of the possible theoretical extensions is to get closer to reality and to study the preference for collusion by assuming a higher marginal cost for the public operator, with a technological gap in comparison the operator private.

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Appendix 1

	Mixed Duopoly	
	Linear Costs	Quadratic Costs
Cournot Profit	$\pi_1^{Comp} = (1-q_{12}-q_{21}\text{-a})\, q_{12}+(a-\mu)q_{21}$ $\pi_2^{Comp} = \frac{1}{2}(q_{12}+q_{21})^2 + (1-q_{12}-q_{21}-a)q_{21} + (a-\mu)q_{12}$	$\pi_1^{Comp} = (1-q_{12}-q_{21}-\text{a})\, q_{12}+(a-2\mu q_{21})q_{21}$ $\pi_2^{Comp} = \frac{1}{2}(q_{12}+q_{21})^2 + (1-q_{12}-q_{21}-a)q_{21} + (a - 2\mu q_{12})q_{12}$
Cournot Quantity	$q_{12} = \frac{1}{2}(1-q_{21}-a); q_{21} = (1-a)$	$q_{12} = \frac{1}{2}\left((1-a)-q_{21}\right);\ \ q_{21} = (1-a)$
Collusion Profit	$\pi_M = \frac{1}{2}Q_M^2 + (1-Q_M-a)Q_M + (a-\mu)Q_M$	$\pi_M = \frac{1}{2}Q_M^2 + (1-Q_M-a)Q_M + (a-2\mu Q_M)Q_M$
Collusion Quantity	$Q_M^* = (1-\mu)$ et $\pi_M^* = \frac{1}{2}(1-\mu)^2$ $q_{12}^{col} = q_{21}^{col} = \frac{1}{2}(1-\mu)$	$Q_M^* = \frac{1}{(1+4\mu)}$; $q_{12}^{col} = q_{21}^{col} = \frac{1}{2(1+4\mu)}$
Deviation Profit	$\pi_1^{\mathrm{Dev}} = (1-q_{12}^{dev}-q_{21}^{col}-a)q_{12}^{dev} + (a-\mu)q_{21}^{col}$	$\pi_1^{\mathrm{Dev}} = (1-q_{12}^{dev}-q_{21}^{col}-a)q_{12}^{dev} + (a-2\mu q_{21}^{col})q_{21}^{col}$
Deviation Quantity	$q_{12}^{dev} = \frac{1}{4}(1+\mu-2a)$	$q_{12}^{dev} = \frac{1}{2}\left((1-a)-q_{21}^{col}\right)$; $q_{12}^{dev} = \frac{1}{2}\left(\frac{1+8\mu}{2+8\mu}-a\right)$

	Private Duopoly	
	Linear Costs	Quadratic Costs
Cournot Profit	$\pi_1 = (1-(q_{12}+q_{21})-a)q_{12} + (a-\mu)q_{21}$ $\pi_2 = (1-(q_{12}+q_{21})-a)q_{21} + (a-\mu)q_{12}$	$\pi_1 = (1-(q_{12}+q_{21})-a)q_{12} + (a-2\mu q_{21})q_{21}$ $\pi_2 = (1-(q_{12}+q_{21})-a)q_{21} + (a-2\mu q_{12})q_{12}$
Cournot Quantity	$q_{12}(q_{21}) = \frac{1}{2}(1-q_{21}-a)$ $q_{21}(q_{12}) = \frac{1}{2}(1-q_{12}-a)$	$q_{12}(q_{21}) = \frac{1}{2}(1-q_{21}-a)$; $q_{21}(q_{12}) = \frac{1}{2}(1-q_{12}-a)$
Collusion Profit	$\pi_M = (1-Q_M-a)Q_M + (a-\mu)Q_M$	$\pi_M = (1-Q_M-a)Q_M + (a-2\mu Q_M)Q_M$
Collusion Quantity	$Q_M^* = \frac{1}{2}(1-\mu)$	$Q_M^* = \frac{1}{2(1+2\mu)}$,
Deviation profit	$\pi_1^{\mathrm{Dev}} = (1-q_{12}^{dev}-q_{21}^{col}-a)q_{12}^{dev} + (a-\mu)q_{21}^{col}$	$\pi_1^{\mathrm{Dev}} = (1-q_{12}^{dev}-q_{21}^{col}-a)q_{12}^{dev} + (a-2\mu q_{21}^{col})q_{21}^{col}$
Deviation Quantity	$q_{12}^{\mathrm{dev}} = \frac{1}{8}(3+\mu-4\mathrm{a})$.	$q_{12}^{dev} = \frac{3+8\mu(1-a)-4a}{8(1+2\mu)}$