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Convex Combination Method on Probabilistic Fuzzy Multiobjective Transportation Problem with Pareto Distribution

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Abstract

This article introduces a transportation model with two objective functions. The first objective function is the function which minimizes the total cost and the second objective function is the function which minimizes the total time. Parameters of source, destination and maximum capacity of transportation means are assumed to follow the Pareto distribution. The multiobjective transportation model is the development of a single objective transportation model. Parameters of the objective function are expressed by triangular fuzzy numbers. Fuzzy multi objective problems are transformed into a deterministic single objective using the convex combination method. The formulated model is applied to the problem of shipping metal crates. There are 3 types of conveyances namely HDL, Engkel and Wingbox. Obtained the optimal total cost of Rp. 3,770,294 and metal crates delivery time to be for 13 hours.

Keywords: convex combination, multiobjective, transportation, pareto distribution.

1. Introduction

Transportation activity is a movement activity associated with traveling from one place to another. Transportation problems are related to transportation problems in order to find optimal cost planning from source to destination which is influenced by the distance of delivery, the number of goods to be sent, the length of time for delivery and so on (Adhami & Ahmad, 2020). In general, the purpose of transportation is to minimize the total cost or total transportation time (Chen et al., 2020), (Tang, 2020), . In some cases, transportation problems have more than one objective function, for example the time and cost functions will be optimized simultaneously. Transportation problems with more than one objective function are called multiobjective transportation problems (Kakran & Dhodiya, 2020), (Bagheri et al., 2020), (Gowthami & Prabakaran, 2019), (Samanta et al., 2020), (Roy et al., 2018).

In transportation problems, there is some information that uncertainty. For example, time data and transportation cost data. The probabilistic fuzzy approach can be used to solve transportation problems with uncertainty and follow certain statistical probability distributions. Research (Kakran & Dhodiya, 2020) provides an assumption that the parameters of the transportation model follow a normal distribution. If the parameters of the transportation model do not follow a certain opportunity distribution then a stochastic approach can be used (Singh et al., 2019), (Boumediene, 2019). Other probability distributions that are often used are the exponential probability distribution (Fergany & Hollah, 2018) and the Pareto distribution (Barik, 2015). In this study, it is assumed that the constraint parameters follow the Pareto distribution. In addition to the probabilistic approach, the fuzzy approach can also be used for parameter uncertainty problems. Model parameters can be expressed as triangular fuzzy numbers (Susanti et al., 2019), hexagonal (Chakraborty et al., 2021), trapezoidal (S & S, 2020), pentagonal (Hemalatha & Annadurai, 2020). In this study, the triangular fuzzy numbers are used.

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Multi-objective problem can be solving by transforming into a single objective form. The convex combination method can be used. Convex combination is a concept programming method used to achieve Pareto optimal solutions of uncertain multi objective programming problems (Dalman, 2018), (Samanta et al., 2020). The optimal solution with the convex combination method is influenced by the determination of the weight values in the objective function. The combination of selecting different weight values gives a different optimal solution. Using this method, decision making has several references as a determinant of optimal policy. Other, the fuzzy programming technique can also be used to solve multiobjective problems (Kakran & Dhodiya, 2020). Matrix concepts and nonlinear programming approaches can be used to solve multiobjective problems (Roy et al., 2018), (Waliv et al., 2020). In this study, multiobjective problems were transformed into a single objective using the convex combination method where a defuzzification process using alpha-cut. The formulated probabilistic fuzzy multi objective solid transportation model is applied to the problem of transporting metal crates. Metal crate is a tool made of iron, useful as a container or storage place for synthetic rubber and natural rubber before the rubber reaches consumers, especially rubber factories around the world. there are three types of vehicles, namely HDL, engkel, and wingbox for the delivery of metal crates. Cost and delivery time data are expressed by triangular fuzzy numbers.

2. Method

The transportation problem is a problem of determining the total minimum transportation cost of a commodity with the aim of meeting demand, generally related to shipping an item from source to destination (Adhami & Ahmad, 2020). (Kakran & Dhodiya, 2020) introduced a multi objective probabilistic fuzzy solid transportation model as follows:

$$\text{Minimum } Z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}x_{ijk}$$
$$\text{Minimum } Z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{t}_{ijk}x_{ijk}$$

Constraints:

$$\begin{aligned}
&\Pr\left(\sum_{j=1}^{n}\sum_{k=1}^{l}x_{ijk} - a_i \le 0\right) \ge \gamma_i \quad, i = 1,2,\dots,m \\
&\Pr\left(b_j - \sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk} \le 0\right) \ge \beta_j \quad, j = 1,2,\dots,n \\
&\Pr\left(\sum_{i=1}^{m}\sum_{j=1}^{n}x_{ijk} - e_k \le 0\right) \ge \delta_k \quad, k = 1,2,\dots,l \\
&x_{ijk} \ge 0 \quad, \forall\, i,j,k
\end{aligned} \tag{1}$$

where:

$\tilde{c}_{ijk}$: unit fuzzy cost source *i* to destination *j* with vehicle *k*
x_{ijk} : amount of product to transport from source *i* to destination *j* with vehicle *k*
$\tilde{t}_{ijk}$: unit fuzzy time source *i* to destination *j* with vehicle *k*
a_i : stock level source *i*
b_j : demand destination *j*
e_k : maximum capacity vehicle *k*
γ_i : level confidence of source constraint *i*
β_j : level confidence of destination constraint *j*
δ_k : level confidence of vehicle constraint *k*
m : source
n : destination
l : type of vehicle

Objective function parameters and constraints are expressed by fuzzy numbers. Zadeh in (Zimmermann, 2010) defines the following fuzzy numbers.

$$\widetilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\} \tag{2}$$

$\mu_{\tilde{A}}(x)$ is called the membership function and lies in the interval [0,1]. The membership degree value can be determined by means of a triangular curve representation. The triangular curve representation is a combination of rising linear representation and descending linear representation. The triangular fuzzy number is changed to a deterministic form using the alpha cut method.

$$r_i = (r_i - \omega_i, r_i, r_i + \omega_{i+1}) \tag{3}$$

with ω_i , i = 1,2,3,... are positif numbers and as follow $0 \leq \omega_i \leq r_i, \omega_{i+1} \geq 0$

$$\tilde{r}_i = r_i + \frac{1}{4}(\omega_{i+1} - \omega_i) \tag{4}$$

Multiobjective problems can be transformed into a single objective form using the convex combination method.

$$\text{Minimum } \sum_{i=1}^{m} \lambda_i E\,[f_i(x,\xi)] \tag{5}$$

constraint:

$$M\{g_j(x,\xi) \leq 0\} \geq \alpha_j, \quad j = 1,2,\ldots,p$$

Weigted value $\lambda_1, \lambda_2, \ldots, \lambda_q$ are positif number in interval [0,1] where $\lambda_1 + \lambda_2 + \cdots + \lambda_q = 1$.

3. Result and Discussion

Statistical distributions that discuss the survival of an object or individual in certain operational conditions are life test data. This analysis can be applied to the field of production related to modeling the survival of a production object in the presence of a limitation on the use of resources to produce a particular product. Reliability is referred to as the survival of production objects. In the analysis of life test data, especially for reliability, the Pareto distribution is often used. The following is the Probability Density Function (PDF) form of the Pareto distribution for two parameters (Barik, 2015).

$$f(b_i) = \frac{p_i (q_i)^{p_i}}{b_i^{(p_i+1)}} \tag{6}$$

by using Equation (6) it can be determined the average and the variance of the PDF, as follows:

$$\bar{b}_i = \frac{p_i\, q_i}{p_i - 1} \quad p_i > 1, i = 1,2,\ldots,m \tag{7}$$

and

$$Var(b_i) = \frac{p_i\, q_i^2}{(p_i-1)^2 (p_i-2)} \quad p_i > 2, i = 1,2,\ldots,m \tag{8}$$

where:

$\bar{b}_i$: *mean*
$Var(b_i)$: variance
p_i = shape parameter *i*th
q_i = scale parameter *i*th

using (2) to (8) then Model (1) can be written as follows.

$$\text{Minimum } Z_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk} \tag{9}$$

$$\text{Minimum } Z_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} t_{ijk} x_{ijk}$$

Constraints:

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}} \quad , i = 1,2,\dots,m$$
$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge \frac{q_i}{(1-\beta_j)^{\frac{1}{p_i}}} \quad , j = 1,2,\dots,n$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le \left(\frac{q_i}{(1-\delta_k)^{\frac{1}{p_i}}}\right)7 \quad , k = 1,2,\dots,l$$

Model (9) is transformed into a single objective form using the convex combination method (2). Further can be written as follows.

$$\text{Min } Z = \left(\lambda_1 \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} c_{ijk}x_{ijk} + \lambda_2 \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} t_{ijk}x_{ijk}\right) \tag{10}$$

constraints:

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} \le \frac{q_i}{(1-\gamma_i)^{\frac{1}{p_i}}} \quad , i = 1,2,\dots,m$$
$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} \ge \frac{q_i}{(1-\beta_j)^{\frac{1}{p_i}}} \quad , j = 1,2,\dots,n$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \le \left(\frac{q_i}{(1-\delta_k)^{\frac{1}{p_i}}}\right)7 \quad , k = 1,2,\dots,l$$

Then the model obtained (9) and (10) is applied to the metal crate transport problem. The objective function parameter values are given in Table 1.

Inventory of metal crates at PT.Elang Marin Sentosa is 23.071. Demand for each destination respectively 160, 180, 195, 120, 160, 150, 110, 55, and 110 metal crates. As well as the payload capacity of each vehicle respectively 70, 120 and 144 metal crates.

Given γ_1 = 0.01, β_1 = 0.06, β_2 = 0.05, β_3 = 0.05, β_4 = 0.04, β_5 = 0.06, β_6 = 0.05, β_7 = 0.06, β_8 = 0.04, β_9 = 0.05, δ_1 = 0.05, δ_2 = 0.06, δ_3 = 0.07 and $\bar{b}_1$ = 13367.25; $\bar{b}_2 = 262.5$; $\bar{b}_3$ = 300; $\bar{b}_4$ = 267; $\bar{b}_5 = 230$; $\bar{b}_6$ = 303.7; $\bar{b}_7 =$ 283; $\bar{b}_8$ = 287.5; $\bar{b}_9$ = 109.2; $\bar{b}_{10}$ = 187.4; $\bar{b}_{11}$ = 57.2; $\bar{b}_{12}$ = 95.8; $\bar{b}_{13}$ = 139.3; q_1 = 6802; q_2= 160; q_3 = 180; q_4= 195; q_5= 120; q_6= 160; q_7= 150; q_8= 110; q_9= 55; q_{10}= 110; q_{11}= 46; q_{12}= 75; q_{13}= 122.

based on the value and using (7) obtained value of shape parameters.

p_1 = 2.04; p_2 = 2.56; p_3 = 2.5; p_4 = 3.71; p_5 = 2.1; p_6 = 2.1; p_7 = 2.13; p_8 = 1.62; p_9 = 2.01; p_{10} = 2.42 p_{11} = 5.1; p_{12} = 4.61; p_{13} = 8.04.

by taking $\lambda_1 = \lambda_2 = 0.5$,

$$\begin{aligned}\text{Min } Z &= 0{,}5(4586x_{111} + 3092x_{112} + \ldots + 3092x_{192} + 3271x_{193}) \\ &\quad +0{,}5(0{,}857x_{111} + 0{,}708x_{112} + \ldots + 0{,}171x_{192} + 0.694x_{193})\end{aligned} \tag{11}$$

Constraints:

$$x_{111} + x_{112} + \cdots + x_{192} + x_{193} \le 6835.659$$
$$x_{111} + x_{112} + x_{113} \ge 163.91282$$
$$x_{121} + x_{122} + x_{123} \ge 183.73126$$
$$x_{131} + x_{132} + x_{133} \ge 197.7159606$$
$$x_{141} + x_{142} + x_{143} \ge 122.3658$$
$$x_{151} + x_{152} + x_{153} \ge 164.7539$$
$$x_{161} + x_{162} + x_{163} \ge 154.425927$$
$$x_{171} + x_{172} + x_{173} \ge 114.2834$$
$$x_{181} + x_{182} + x_{183} \ge 56.12539$$
$$x_{191} + x_{192} + x_{193} \ge 112.35523$$

$$x_{111} + x_{121} + \cdots + x_{181} + x_{191} \leq 325.2554934$$
$$x_{112} + x_{122} + \cdots + x_{182} + x_{192} \leq 532.0946298$$
$$x_{113} + x_{123} + \cdots + x_{183} + x_{193} \leq 862.1780799$$

Completion of Model (11) uses Lingo 13.0 and the following solutions are obtained.

The application of the convex combination method will provide recommendations for the number of different shipments of metal crates to each destination using 3 vehicles in order to obtain minimum costs and travel time. By taking the weight of the value λ where the value of $\lambda_1 \neq 0, \lambda_2 \neq 1$ and $\lambda_1 + \lambda_2 = 1$, the optimal number of metal crates will be obtained which will be distributed using different transportation vehicles. Based on Model (11), transportation costs and time will be minimized by not using HDL type vehicles.

Table 1. Fuzzy Cost and Time Parameters

Cost Parameters	*TFN*	Alfa Cut	Time Parameters	*TFN*	Alfa Cut
$\widetilde{c_{111}}$	(320, 320,325)	321	$\widetilde{t_{111}}$	(59, 59,63)	60
$\widetilde{c_{112}}$	(370, 370,375)	371	$\widetilde{t_{112}}$	(84, 84,88)	85
$\widetilde{c_{113}}$	(470, 470,475)	471	$\widetilde{t_{113}}$	(97,97,106)	99
$\widetilde{c_{121}}$	(305, 305,310)	306	$\widetilde{t_{121}}$	(54,54,58)	55
$\widetilde{c_{122}}$	(355, 355,360)	356	$\widetilde{t_{122}}$	(79,79,83)	80
$\widetilde{c_{123}}$	(455, 455,460)	456	$\widetilde{t_{123}}$	(92,92,101)	94
$\widetilde{c_{131}}$	(295, 295,300)	296	$\widetilde{t_{131}}$	(47,47,50)	48
$\widetilde{c_{132}}$	(345, 345,350)	346	$\widetilde{t_{132}}$	(71,71,75)	72
$\widetilde{c_{133}}$	(445, 445,450)	446	$\widetilde{t_{133}}$	(84,84,92)	86
$\widetilde{c_{141}}$	(280, 280,285)	281	$\widetilde{t_{141}}$	(44,44,47)	45
$\widetilde{c_{142}}$	(330, 330,335)	331	$\widetilde{t_{142}}$	(69,69,72)	70
$\widetilde{c_{143}}$	(430, 430,435)	431	$\widetilde{t_{143}}$	(82,82,89)	84
$\widetilde{c_{151}}$	(315,315,320)	316	$\widetilde{t_{151}}$	(59,59,61)	62
$\widetilde{c_{152}}$	(365, 365,370)	366	$\widetilde{t_{152}}$	(84,84,86)	84
$\widetilde{c_{153}}$	(465, 465,470)	466	$\widetilde{t_{153}}$	(96,96,104)	98
$\widetilde{c_{161}}$	(230, 230,235)	231	$\widetilde{t_{161}}$	(28,28,29)	28
$\widetilde{c_{162}}$	(280, 280,285)	281	$\widetilde{t_{162}}$	(53,53,54)	53
$\widetilde{c_{163}}$	(380, 380,385)	381	$\widetilde{t_{163}}$	(66,66,75)	68
$\widetilde{c_{171}}$	(260,260,265)	261	$\widetilde{t_{171}}$	(39,39,45)	41
$\widetilde{c_{172}}$	(310, 310,315)	311	$\widetilde{t_{172}}$	(64,64,70)	66
$\widetilde{c_{173}}$	(410, 410,415)	411	$\widetilde{t_{173}}$	(77,77,90)	80
$\widetilde{c_{181}}$	(255, 255,260)	256	$\widetilde{t_{181}}$	(39,39,41)	40
$\widetilde{c_{182}}$	(305, 305,310)	306	$\widetilde{t_{182}}$	(64,64,66)	65
$\widetilde{c_{183}}$	(405, 405,410)	406	$\widetilde{t_{183}}$	(77,77,84)	79
$\widetilde{c_{191}}$	(320, 320,325)	321	$\widetilde{t_{191}}$	(61,61,65)	62
$\widetilde{c_{192}}$	(370, 370,375)	371	$\widetilde{t_{192}}$	(85,85,90)	86
$\widetilde{c_{193}}$	(470, 470,475)	471	$\widetilde{t_{193}}$	(98,98,105)	100

Source: PT.Elang Marin Sentosa South Sumatera

4. Conclusion

Based on the results and discussion, it is concluded that the convex combination method can be used to change multiobjective transportation problems to single objective problems. Variations in values taken as multiplier weights in convex combination method affect the optimal solution obtained. For application to metal crates shipping problems, the optimal cost and time obtained are Rp. 3,770,294 and 13 hours, respectively. The optimal number of metal crates to transport using the Engkel is 40 metal crates to the fourth destination, 165 metal crates to the fifth destination, 155 metal crates to the sixth destination, 115 metal crates to the seventh destination, 57 metal crates to the eighth destination. Using wingbox 164 metal crates to the first destination, 184 metal crates to the second destination, 198 metal crates to the third destination, 83 metal crates to the fourth destination, and 113 metal crates to the ninth destination.

Table 2. Optimal Solution

Variable	Solution	Variable	Solution
x_{111}	0	x_{153}	0
x_{112}	0	x_{161}	0
x_{113}	164 *metal crates*	x_{162}	155 *metal crates*
x_{121}	0	x_{163}	0
x_{122}	0	x_{171}	0
x_{123}	184 *metal crates*	x_{172}	115 *metal crates*
x_{131}	0	x_{173}	0
x_{132}	0	x_{181}	0
x_{133}	198 *metal crates*	x_{182}	57 *metal crates*
x_{141}	0	x_{183}	0
x_{142}	40 *metal crates*	x_{191}	0
x_{143}	83 *metal crates*	x_{192}	0
x_{151}	0	x_{193}	113 *metal crates*
x_{152}	165 *metal crates*	x_{153}	0
Z_1	Rp 3.770.294	Z_2	12,9824 hours

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