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Exploring Nano Simply Open Sets in Nano Topology: Applications and Insights

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Abstract

Nano topology is a modern type of topology based on rough sets, introducing nano open sets and their properties. In recent times, nano set theory has shown broad applications in addressing practical challenges in various domains, including engineering, social sciences, and healthcare sciences. The notion of nano topological spaces, along with associated concepts like nano near open sets, has greatly developed because of its practical usefulness. The idea of nano continuity extends the idea of continuity. Likewise, nano open sets extend the notion of open sets within topological spaces. This paper presents a new class of nano near open sets, termed "nano simply open sets" and "nano delta sets," which expands and alters the current concepts of nano open sets (and, in certain instances, nano near open sets). We introduce a novel category of nano simply open sets, explore their characteristics, and analyze their connections with other sets. Additionally, we investigate new ideas like "nano simply continuous functions" and also, we introduce new notions of continuity based on the new notions. We shall study some of their properties and We will study the relationship between the new concepts and various other nano open sets.

Keywords: nano open set nano delta open set, nano simply open set, mightily nano simply continuous, and nano locally indiscrete space.

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1. Introduction

Nano topology, introduced by Thivagar et al. (2022), extends classical set theory to address intelligent systems dealing with incomplete and insufficient information. In nano topological spaces, elements are described as nano open sets. The concept was formulated using lower and upper approximations, and the authors proposed weaker various types of nano open sets, including nano semi-open sets and nano pre-open sets. Additionally, they extended the notion of continuity by Thivagar et al. (2022) and El Sayed et al. (2024), a fundamental concept in topology, to nano topological spaces. Thivagar et al. (2022) further defined the nano continuous function, Thivagar etal, expanded on this theory and introduced rough topology, which they called nano topology.

Nano topology is a modern type of topology based on rough sets, with elements referred to as nano open sets. The term "nano" here signifies "very small," originating from the Greek word "Nanos," meaning dwarf. The nano topology has at most five elements and includes nano closed sets, nano interior, and nano closure. Additionally, introduced weak forms of nano open sets such as nano α-open sets, nano semi-open sets, nano pre-open sets, and nano regular open sets. El-Bably & El Sayed (2018), extends the concept of nano topology to include nano ordered topology, which involves nano-increasing or decreasing topological spaces. On the other hand, Khalifa & Jasim (2021), introduces the concept of nano dense via graph theory. These structures offer distinct equations for various sets and provide enhanced solutions for decision-making issues. They find diverse applications in fields including medical diagnosis, pattern recognition, social sciences, artificial intelligence, business, and multi-attribute decision-making problems. For example, Alshammari et al. (2021), explore multiple attribute decision-making by fuzzy nano topological spaces; El-Atik et al. (2023), apply the concept of fuzzy soft sets and decision-making in ideal nutrition, while other researchers by Mohsin et al. (2023), introduced new types of mappings. In traditional topology, open sets form a foundational concept, which can be generalized to nano and soft topological spaces. These generalized sets are essential in various applications across diverse fields, as noted in see Alblowi et al. (2021).

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This paper presents the ideas of "nano simply open sets and "nano delta sets" as extensions of existing nano near open sets, exploring their properties and relationships with other nano sets. Moreover, new continuous functions, including "nano simply continuous," "mightily nano simply continuous," and "nano simply irresolute functions," are defined and analyzed. The paper also investigates connections between different nano open sets, enhancing our understanding of nano topological spaces and their potential applications to real-world challenges.

Explore medical applications in nano-topological space; and discuss the medical applications of Couroupita Guianese's Abul plant in the context of COVID-19. Also, researchers in Khalil & Abbas (2020), Jasim et al. (2021), El Sayed et al. (2021), Jayalakshmi & Janaki (2017) and Hu et al. (2022).

Definition 1.1. Pawlak (1982), A twofold relationship $\wp$on a set $\Bbbk$ is said to be:

1. *Reflexive: if for each* $\mathfrak{Z} \in \Bbbk, \mathfrak{Z}\wp\mathfrak{Z}$.
2. *Symmetrical: if for each* $\mathfrak{Z}, \omega \in \Bbbk$ and $\mathfrak{Z}\wp\omega$, then $\omega\wp\mathfrak{Z}$.
3. *Transitive*: if for all $\mathfrak{Z}, \omega, \mathscr{b} \in \Bbbk$, $\mathfrak{Z}\wp\omega$ and $\omega\wp\mathscr{r}$, then $\mathfrak{Z}\wp\mathscr{r}$.
4. *Resemblance or acceptance*: if $\mathscr{r}$ is both symmetric and reflexive.
5. *Pre-order or dominance*: if $\mathscr{r}$ is a reflexive and transitive.
6. If $\mathscr{r}$ is a transitive, symmetric, reflexive relation, then equivalency exists.

Definition 1.2. Pawlak (1982), assuming that $\mathfrak{M}$ is a finite set referred to as a universal set and let $\mathfrak{w}$ denote an equivalence relation on $\mathfrak{M}$, we write $\mathfrak{M}/\mathfrak{w}$ to symbolize the class of every category of equivalency of $\mathfrak{w}$ while $[\mathfrak{D}]_{\mathfrak{w}}$ signifies the equivalence class within in $\mathfrak{w}$ that includes the element $\mathfrak{D} \in \mathfrak{M}$. Subsequently, the pair $(\mathfrak{M}, \mathfrak{w})$ is referred to as an approximation space, and for any given $\mathfrak{L} \subseteq \mathfrak{M}$, We recommend the Pawlak-lower and Pawlak-upper estimates of $\mathfrak{L}$ by: $\mathfrak{w}_*(\mathfrak{L}) = \{\mathfrak{D} \in \mathfrak{M}: [\mathfrak{D}]_{\mathfrak{w}} \subseteq \mathbb{A}\}$ and $\mathfrak{w}^*(\mathbb{A}) = \{\mathfrak{D} \in \mathfrak{M}: [\mathfrak{D}]_{\mathfrak{w}} \cap \mathfrak{L} \neq \emptyset\}$, respectively. Accordingly, we define the following regions:

1. The border area of $\mathfrak{L} \subseteq \mathfrak{M}$: $\mathrm{BND}_{\mathfrak{w}}(\mathfrak{L}) = \mathfrak{w}^*(\mathfrak{L}) - \mathfrak{w}_*(\mathfrak{L})$.
2. The positive area of $\mathfrak{L} \subseteq \mathfrak{M}$: $\mathrm{POS}_{\mathfrak{w}}(\mathfrak{L}) = \mathfrak{w}_*(\mathfrak{L})$.
3. The negative area of $\mathfrak{L} \subseteq \mathfrak{M}$: $\mathrm{NEG}_{\mathfrak{w}}(\mathfrak{L}) = \mathfrak{M} - \mathfrak{w}^*(\mathfrak{L})$.

The accurate of Pawlak's approximation is: $\mu_{\mathfrak{w}}(\mathfrak{L}) = \frac{|\mathfrak{w}_*(\mathfrak{L})|}{|\mathfrak{w}^*(\mathfrak{L})|}$, in which $\mathfrak{w}^*(\mathfrak{L}) \neq \varphi$.

Proposition 1.1. Pawlak (1982), let's assume that φ denotes a null set and $\mathfrak{L}^c$ is a complement of $\mathfrak{L}$ in $\mathfrak{M}$. For each $\mathfrak{L}, \mathfrak{T} \subseteq \mathfrak{M}$, the next items are maintained:

(L1) $\mathfrak{w}_*(\mathfrak{L}) \subseteq \mathfrak{L}$
(L2) $\mathfrak{w}_*(\varphi) = \varphi$
(L3) $\mathfrak{w}_*(\mathfrak{M}) = \mathfrak{M}$
(L4) $\mathfrak{w}_*(\mathfrak{L} \cap \mathfrak{T}) = \mathfrak{w}_*(\mathfrak{L}) \cap \mathfrak{w}_*(\mathfrak{T})$
(L5) If $\mathfrak{L} \subseteq \mathbb{B}$, *then* $\mathfrak{w}_*(\mathfrak{L}) \subseteq \mathfrak{w}_*(\mathfrak{T})$
(L6) $\mathfrak{w}_*(\mathfrak{L}) \cup \mathfrak{w}_*(\mathfrak{T}) \subseteq \mathfrak{w}_*(\mathfrak{L} \cup \mathfrak{T})$
(L7) $\mathfrak{w}_*(\mathfrak{L}^c) = (\mathfrak{w}^*(\mathfrak{L}))^c$
(L8) $\mathfrak{w}_*(\mathfrak{w}_*(\mathfrak{L})) = \mathfrak{w}_*(\mathfrak{L})$
(L9) $\mathfrak{w}_*((\mathfrak{L}) = (\mathfrak{w}_*(\mathfrak{L}))^c$

(U1) $\mathfrak{L} \subseteq \mathfrak{w}^*(\mathfrak{L})$
(U2) $\mathfrak{Y}^*(\varphi) = \varphi$
(U3) $\mathfrak{w}^*(\mathfrak{M}) = \mathfrak{M}$
(U4) $\mathfrak{w}^*(\mathfrak{L} \cup \mathfrak{T}) = \mathfrak{w}^*(\mathfrak{L}) \cup \mathscr{b}^*(\mathfrak{T})$
(U5) If $\mathfrak{L} \subseteq \mathfrak{T}$, *then* $\mathfrak{w}^*(\mathfrak{L}) \subseteq \mathfrak{w}^*(\mathfrak{T})$
(U6) $\mathfrak{w}^*(\mathfrak{L}) \cap \mathfrak{w}^*(\mathfrak{T}) \supseteq \mathfrak{w}^*(\mathfrak{L} \cap \mathfrak{T})$
(U7) $\mathfrak{w}^*(\mathfrak{L}) = (\mathfrak{w}_*(\mathfrak{L}))^c$
(U8) $\mathfrak{w}^*(\mathfrak{w}^*(\mathfrak{L})) = \mathfrak{w}^*(\mathfrak{L})$
(U9) $\mathfrak{w}^*((\mathfrak{w}^*(\mathfrak{L}))^c) = (\mathfrak{w}^*(\mathfrak{L}))^c$

Remark 1.1: The characteristics (*L9*) and (*U9*) signify that the lower and upper approximations refer to precise sets, whereas the characteristics (*L10*) and (*U10*) demonstrate that the components in $\mathfrak{M}/\mathfrak{w}$, are precise sets.

Definition 1.3 Thiyagar et al. (2022), let $\mathfrak{M}$ be the universe, $\mathfrak{w}$ be represented by as an equivalence relation on $\mathfrak{M}$ and$\mathfrak{T}_{\mathfrak{w}}(\aleph) = \{\mathfrak{M}, \varphi, \mathfrak{w}_*(\aleph), \mathfrak{w}^*(\aleph), \mathrm{BND}_{\mathfrak{w}}(\aleph)\}$, where $\aleph \subseteq \mathfrak{M}$.

Definition 1.4 Thiyagar et al. (2022), let $(\mathfrak{M}, \mathfrak{T}_R(\aleph))$ be a nano space, $\mathfrak{I} \subseteq \mathfrak{M}$. Subsequently $\mathfrak{I}$ it is referred to as:

1. A nano semi-open if $\mathfrak{I} \subseteq Cl(Int(\mathfrak{I}))$.
2. Nano pre-open if $\mathfrak{I} \subseteq Int(Cl(\mathfrak{I}))$.
3. Nano $\alpha - open\ if\ \mathfrak{I} \subseteq Int(Cl(Int(\mathfrak{I})))$.
4. Nano regular open if $\mathfrak{I} = Int(Cl(\mathfrak{I}))$.

Definition 1.5 Revathy & Ilango (2015), let $(\mathfrak{M}, \mathfrak{T}_{\mathscr{b}}(\aleph))$ be a nano space and $\mathfrak{I} \subseteq \mathfrak{M}$. Subsequently $\mathfrak{I}$ it is called nano β-open in $\mathfrak{M}$ if $\mathfrak{I} \subseteq cl^N(int^N(cl^N(\mathfrak{I})))$. The assembly of all nano β-open sets of $\mathfrak{M}$ is represented through, $\beta^N O(\mathfrak{M})$.

Definition 1.6 Thiyagar *etal*, let $(\mathfrak{M}, \mathfrak{T}_{\mathfrak{w}}(\aleph))$ be a nano space and $\aleph \subseteq \mathfrak{M}$. Subsequently.

1. The set's nano interior $\mathbb{S}$ is the union of all nano open subsets is what it means to be enclosed in $\mathbb{S}$ and is represented by $(int^N$ $(\mathbb{S})$ is the biggest nano open.
2. The nanoscopic closure of set $\mathbb{S}$ is described as the intersection of all nano closed. sets including $\mathbb{S}$ and is described as $cl^N(\mathbb{S})$ is the tiniest closed nano set that includes $\mathbb{S}$.

Definition 1.7 Thiyagar et al. (2022), let $\mathfrak{M}$ be the universe, $\mathfrak{w}$ To qualify as an equivalence relation on $\mathfrak{M}$ and $\mathfrak{T}_{\mathfrak{w}}(\aleph) = \{\mathfrak{M}, \varphi, \mathfrak{w}_*(\aleph), \mathfrak{w}^*(\aleph), \mathrm{BND}_{\mathfrak{w}}(\aleph)\}$ where $\aleph \subseteq \mathfrak{M}$.

where $\aleph \subseteq \mathfrak{M}$. Subsequently $\mathfrak{T}_{\mathfrak{w}}(\aleph)$ fulfills the subsequent axioms:

1. U and $\emptyset \in \mathfrak{T}_{\mathfrak{w}}(\aleph)$.
2. The combination of the components of any sub-collection of $\mathfrak{T}_{\mathfrak{w}}(\aleph)$ is in $\mathfrak{T}_{\mathfrak{w}}(\aleph)$.
3. The common components among any selected subset of finite sets of $\mathfrak{T}_{\mathfrak{w}}(\aleph)$ is in $\mathfrak{T}_{\mathfrak{w}}(\aleph)$.

Then $\mathfrak{T}_{\mathfrak{w}}(\aleph)$ is a topology on $\mathfrak{M}$ referred to as the nano topology on $\mathfrak{M}$ with respect to $\aleph$.

$(\mathfrak{M}, \mathfrak{T}_R(\aleph))$ This is referred to as nano topological space. The elements within this space are termed nano open sets in $\mathfrak{M}$. The complement of a nano open set is recognized as a nano closed set. $(\mathfrak{T}_R(\aleph))^c$ are referred to as dual nano topology.

Definition 1.8 Nachiyar & Bhuvaneswari (2014), A nano function $f: (\mathfrak{U}, \mathrm{T}_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, \mathrm{T}_{\mathfrak{w}}((\mathfrak{Z}))$ it is referred to as nano continuous if for every nano open set $\mathfrak{W}$ in $\mathfrak{V}$ (resp., nano closed $\mathfrak{Q}$) of $\mathfrak{V}$, $f^{-1}(\mathfrak{W})$ (resp. $f^{-1}(\mathfrak{Q})$) is nano open in $\mathfrak{U}$.

2. Nano simply open sets

This section introduces new concepts within nano topological spaces, including nano nowhere dense sets, nano simply open sets, nano delta sets, as well as nano semi-locally closed sets. Their properties are examined in detail, along with the relationships between these concepts. To clarify and demonstrate these connections, various examples and counterexamples are provided.

Definition 2.1. A nano subset $\wp$ of a of nano space $(\mathfrak{W}, \mathfrak{T}_{\mathscr{b}\mathfrak{w}}(\aleph))$ it goes by the name of nano simply – open set if $int(cl(\wp)) \subseteq cl(int(\wp))$.

Definition 2.2 A nano subset $\mathfrak{F}$ of nano space $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$ it is referred to as:

1. nano nowhere dense if $int(cl(\mathfrak{F})) = \tilde{\varphi}$.
2. nano simply open set if $\mathfrak{F} = \mathfrak{G} \cup \mathfrak{V}$, in which, $\mathfrak{G}$ is nano open set and $\mathfrak{V}$ is nano nowhere dense set.
3. nano delta set (briefly, nano δ -set) if $int(cl((\mathfrak{F})) \subseteq cl(int((\mathfrak{F})))$.
4. nano semi locally closed set if nano set $\mathfrak{F}$ is equal to the junction of nano semi open set and nano semi closed set. We will refer to the category of nano simply open, (resp., nano simply closed set, nano nowhere dense, nano delta, nano delta closed, nano semi open, nano semi closed, nano preopen nano pre closed) sets of a universe sets $\mathfrak{W}$ by $\overset{\mathfrak{N}}{SO}(\mathfrak{W})$.

(resp. $\overset{\mathfrak{N}}{SC}(\mathfrak{W})$, $\mathfrak{N}^{\mathfrak{D}}(\mathfrak{W})$, $\delta^{\mathfrak{N}}O(\mathfrak{W})$, $\delta^{\mathfrak{N}}C(\mathfrak{W}), \mathfrak{S}_nO(\mathfrak{W}), \mathfrak{S}_nC(\mathfrak{W}), \mathfrak{P}_nO(\mathfrak{W}), \mathfrak{P}_nC(\mathfrak{W})$)

Example 2.1 Let $\mathfrak{W} = \{\mathfrak{t}, \mathfrak{p}, \mathfrak{q}, \mathfrak{v}\}$, $\frac{\mathfrak{w}}{\mathfrak{w}} = \{\{\mathfrak{t}\}, \{\mathfrak{p}, \mathfrak{v}\}, \{\mathfrak{q}\}\}$ and $\mathfrak{X} = \{\mathfrak{t}, \mathfrak{p}\}$ then $\overline{\mathfrak{w}\mathfrak{X}} = \{\mathfrak{t}, \mathfrak{p}, \mathfrak{v}\}$, $\underline{\mathfrak{w}}\mathfrak{X} = \{\mathfrak{t}\}$ $\mathscr{b}^{\mathfrak{N}\mathfrak{D}}(\mathfrak{X}) = \{\mathfrak{p}, \mathfrak{v}\}$, $\mathfrak{T}_{\mathfrak{w}}(\mathfrak{X}) = \{\mathfrak{W}, \emptyset, \{\mathfrak{t}\}, \{\mathfrak{t}, \mathfrak{p}, \mathfrak{v}\}, \{\mathfrak{p}, \mathfrak{v}\}\}$, and $\mathfrak{T}_{\mathfrak{w}}{}^c(\mathfrak{X}) = \{\mathfrak{W}, \emptyset, \{\mathfrak{q}\}, \{\mathfrak{t}, \mathfrak{q}\}, \{\mathfrak{p}, \mathfrak{q}, \mathfrak{v}\}\}$, thus the class of nano simply open sets is $S^{\mathfrak{S}}O(\mathfrak{W}) = \{\mathfrak{W}, \emptyset, \{\mathfrak{t}\}, \{\mathfrak{q}\}, \{\mathfrak{t}, \mathfrak{q}\}, \{\mathfrak{p}, \mathfrak{v}\}, \{\mathfrak{t}, \mathfrak{p}, \mathfrak{v}\}, \{\mathfrak{p}, \mathfrak{q}, \mathfrak{v}\}\}$ and the complement of the class of nano simply open sets is $S^{\mathfrak{S}}C(\mathfrak{W}) = \{\emptyset, \mathfrak{W}, \{\mathfrak{p}, \mathfrak{q}, \mathfrak{v}\}, \{\mathfrak{t}, \mathfrak{p}, \mathfrak{v}\}, \{\mathfrak{t}, \mathfrak{q}\}, \{\mathfrak{p}, \mathfrak{v}\}, \{\mathfrak{q}\}, \{\mathfrak{t}\}\}$

Theorem 2.1. For a nano subset $\mathfrak{y} \subseteq \mathfrak{W}$ The conditions listed below are comparable.:

1) $\mathfrak{y}$ is nano simply open set.
2) $\mathfrak{y}$ is nano semi-locally closed set.
3) $\mathfrak{y}$ is a nano δ -set.
4) $\mathfrak{y}$ is nano nowhere dense set.

Proof. (1) $\rightarrow$ (2) evident.

(2) $\rightarrow$ (3) Permit $\mathfrak{y}$ be nano semi locally closed. Then $int(cl(\mathfrak{y}))) \subseteq int(cl(\mathfrak{y})) \cap cl(sint(\mathfrak{y}))$ and $int(cl(\mathfrak{y})) \subseteq cl(int(\mathfrak{y}))$. Thus $\mathfrak{y}$ is a nano δ -set.

(3) $\rightarrow$ (4) Permit $\mathfrak{y}$ be a nano δ -set, subsequently,

$int(scl(\mathfrak{y})) = int(scl(\mathfrak{y})) \cap int(scl(\mathfrak{W}\backslash\mathfrak{y})) = int(cl(\mathfrak{y})) \cap \left(\mathfrak{W}\backslash cl(sint(\mathfrak{y}))\right) = int(scl(\mathfrak{y}))\backslash\, cl(sint(\mathfrak{y})) = \varphi.$

(4) $\rightarrow$ (1) evident.

Theorem 2.2 In any nano space, the complementary of the class of nano simply open sets are nano simply open set i.e. $\overset{\mathfrak{N}}{\mathrm{SO}}(\mathfrak{W}) = \overset{\mathfrak{N}}{\mathrm{SC}}(\mathfrak{W})$.

Proof. Let $\mathfrak{V}$ is nano simply open set , then from the above theorem $\mathfrak{V}$ is nano δ -set and hence we have, $int(cl(\mathfrak{V})) \subseteq cl(sint(\mathfrak{V}))$. Taking the complement of each side results in, $\mathfrak{W}\backslash cl(sint(\mathfrak{V})) \subseteq \mathfrak{W}\backslash int(cl(\mathfrak{V}))$ this tends to $int(cl((\mathfrak{W}\backslash\mathfrak{V})) \subseteq cl(int((\mathfrak{W}\backslash\mathfrak{V}))$ Therefore $(\mathfrak{W}\backslash\mathfrak{V}) = \mathfrak{V}^c = \mathfrak{V}$.

Theorem 2.3 Let $(\mathfrak{W}, \mathrm{T}_{\mathfrak{w}}(\mathfrak{X}))$be any nano space. Then:

1. The combination of two nano simply open sets is a nano simply open set.
2. The finite intersection of nano simply open sets is also a nano simply open set.

Proof.

1. Permit $\mathfrak{V}$ and $\mathfrak{Y}$ be two nano simply sets, then. $int(cl(\mathfrak{V})) \subseteq cl(int(\mathfrak{V}))$ (1)

   Also, since $\mathfrak{Y}$ is nano simply set. Afterwards $int(cl(\mathfrak{Y})) \subseteq cl(int(\mathfrak{Y}))$ (2)

   $int(cl(\mathfrak{V})) \cup int(cl(\mathfrak{Y})) \subseteq cl(int(\mathfrak{V})) \cup cl(int(\mathfrak{Y})) \subseteq cl(int(\mathfrak{G})) \cup cl(int(\mathfrak{V}))$

Thus, from (1) and (2), we get $int(cl(\mathfrak{V})) \cup int(cl(\mathfrak{Y})) \subseteq cl(int(\mathfrak{V})) \cup cl(int(\mathfrak{Y})) \subseteq cl(int(\mathfrak{Y})) \cup cl(int(\mathfrak{V}))$ this implies that $int(cl((\mathfrak{V})) \cup (\mathfrak{Y}))) \subseteq cl(int((\mathfrak{V})) \cup (\mathfrak{Y})))$ Now, if we put $\mathfrak{V} \cup \mathfrak{Y} = \mathfrak{Z}$, then $int(cl(\mathfrak{Z}) \subseteq cl(int(\mathfrak{Z}))$. Hence, $\mathfrak{Z}$ is nano simply open set.

2. Let $\{\mathfrak{h}\}_{i=1}^{n}$ be a collection of nano simply open sets, then $int(cl(\{\mathfrak{h}\}_{i=1}^{n}) \subseteq cl(int(\{\mathfrak{h}\}_{i=1}^{n}))$ for each i= 1,2, ..., n. Thus $int(cl(\{\mathfrak{h}\}_{i=1}^{n}) \subseteq cl(int(\{\mathfrak{h}\}_{i=1}^{n}))$ and this implies $\bigcap_{i=1}^{n} int(cl(\{\mathfrak{h}\}_{i=1}^{n}) \subseteq \bigcap_{i=1}^{n} cl(int(\{\mathfrak{h}\}_{i=1}^{n}))$. Hence $sint(scl(\bigcap_{i=1}^{n} \{\mathfrak{h}\}_{i=1}^{n}) \subseteq cl(int(\bigcap_{i=1}^{n} \{\mathfrak{h}\}_{i=1}^{n}))$, therefore $\bigcap_{i=1}^{n} \{\mathfrak{h}\}_{i=1}^{n}$ is nano simply open set.

The figure 1 illustrates the connection between nano simply open sets and several other types of nano close to open sets.

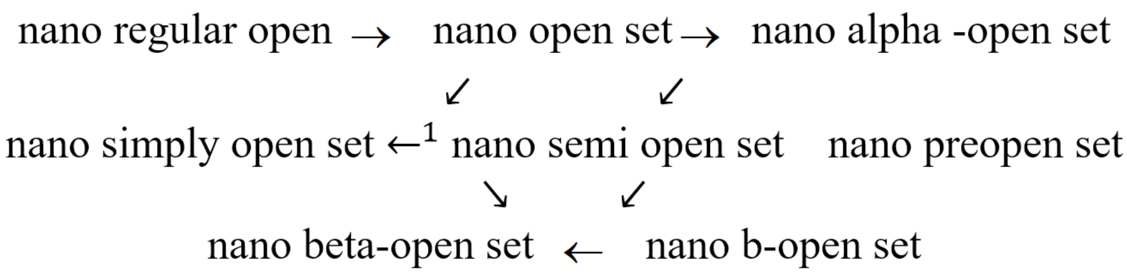

Figure 1. The relations between nano simply and others

Example 2.2 From Example 2.1 we have the set { զ } is nano simply open set but not nano semi open set**.**

Remark 2.1 Evidently, each nano semi-open and every nano semi-closed set is nano simply open. Conversely, not every nano simply open set is nano semi open and nano semi closed sets. The example that follows demonstrates this. remark.

Example 2.3. Permit, $\mathfrak{W} = \{\mathrm{e}, n, y, z\}, \frac{\mathfrak{W}}{\mathfrak{w}} = \{\{\mathrm{e}, z\}, \{n\}, \{y\}\}$, $\mathfrak{X} = \{\mathrm{e}, y\}$then the nano topology on $\mathfrak{W}$ with resect to $\mathfrak{X}$ is $\mathrm{T}_{\mathscr{b}}(\mathfrak{X}) = \{\mathfrak{W}, \emptyset, \{y\}, \{y, z\}, \{\mathrm{e}, y, z\}\}$. Since the sets $\{\mathrm{e}, n, y\}$ and $\{n, y\}$ are nano semi open and it is not semi closed correspondingly, also they are nano simply open sets but the conversely the set $\{n\}$ is nano simply open but it's not semi open.

Remark 2.2 If $(\mathfrak{W}, T_{\mathscr{b}}(\mathfrak{X}))$is nano topological space, then from Figure 1:

1. Nano simply open set and nano β-open set cannot be compared.
2. Nano simply open set and nano b-open set cannot be compared.
3. Nano simply open set and nano preopen set cannot be compared.

The following example shows this remark.

Example 2.4 From Example 2.1.

1. we get the set $\{\mathfrak{v}\}$ is nano beta open but not nano simply open set and the $\{\mathfrak{q}\}$ is nano simply open set but not nano beta open set.
2. The set $\{\mathfrak{v}\}$ is nano b- open but not nano simply open set and the $\{\mathfrak{q}\}$ is nano simply open set but not nano b- open set.
3. The set $\{\mathfrak{t}, \mathfrak{q}, \mathfrak{v}\}$ is nano preopen but not nano simply open set and the $\{\mathfrak{q}\}$ is nano simply open set but not nano preopen set.

Theorem 2.4 In a nano space $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$ if $\mathfrak{E}$ is a nano simply open set and nano regular open. Consequently, $\mathfrak{E}$ is nano semi open set.

Proof. Evident.

Theorem 2.5 In a nano space $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$if $\mathfrak{E}$ is a nano simply open set and nano regular closed. Then $\mathfrak{E}$ is nano semi open closed set.

Proof. Evident.

Theorem 2.6 In a nano space $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$ if $\mathfrak{E}$ is a nano simply open set and nano regular closed. Then $\mathfrak{E}$ is nano semi open closed set.

Proof. Evident.

Theorem 2.7 In a nano space $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$and $\mathfrak{F} \subseteq \mathfrak{W}$ the following conditions are equivalent:

1. $\mathfrak{F}$ is a nano semi-closed set.
2. $\mathfrak{F}$ is nano $sg-$ closed and nano simply open set.

Proof. (1) $\rightarrow$ (2) Let $\mathfrak{F}$ be nano semi closed set and let $\mathfrak{H}$ be nano semi open set. Then $\mathrm{scl}(\mathfrak{F}) \subseteq \mathfrak{F} \subseteq \mathfrak{H}$. Hence $\mathfrak{F}$ is nano semi generalized closed set.

(2) $\rightarrow$ (1) Let $\mathrm{scl}(\mathfrak{F})$ denote the nano semi-closure of $\mathfrak{F}$, i.e. the intersection of all nano semi-closed nano super sets of $\mathfrak{F}$. Since $\mathfrak{F}$ is nano simply open, then $\mathfrak{F}$ can be written as where the nano semi-open set intersects $\mathfrak{H}$ and nano semi-closed set $\mathfrak{V}$. Since $\mathfrak{F}$ is nano sg- closed, we have $\mathrm{scl}(\mathfrak{F}))$ is contained in $\mathfrak{H}$. Since $\mathfrak{V}$ is nano semi-closed, $\mathrm{scl}(\mathfrak{F})$ is contained in $\mathrm{T}_{\mathfrak{w}}((\mathfrak{X}))$. Therefore $\mathrm{scl}((\mathfrak{F})) = \mathfrak{F}$, i.e. $\mathfrak{F}$ is nano semi-closed.

Definition 2.3 A nano space $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$is called nano locally indiscrete space if every nano open set is nano closed set.

Proposition 2.1 For a nano topological space $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$. The criteria listed below are equivalent:

1. Each nano simply open set is nano semi-closed.
2. Each nano open set is a regular nano open set.
3. $(\mathfrak{W}, T_{\mathfrak{w}}(\mathfrak{X}))$ is a locally indiscrete nano space.
4. Every nano simply open set is nano α-closed set.

Proof. Let (1) $\rightarrow$ (2) and $\mathscr{f}$ be nano open. Subsequently, $\mathscr{f}$ is also nano semi-closed and thus nano regular open, and so (2) holds.

(2) $\rightarrow$ (3) Evident.
(3) $\rightarrow$ (4) Let $\not f$ be nano simply open, i.e. $\mathrm{int}(\mathrm{cl}(\not f)) \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(\not f))) \subseteq \mathrm{cl}(\mathrm{int}((\not f))) \subseteq \not f$. By (3), then $\mathfrak{A}$ is nano α-closed.
(4) $\rightarrow$ (1) let $\not f$ be nano α - closed set and nano simply open set. subsequently, $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(\not f) \subseteq \not f$ but $\mathrm{int}(\mathrm{cl}(\not f)) \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(\not f))) \subseteq \mathrm{cl}(\mathrm{int}((\not f)) \subseteq \not f$ this means which, $\not f$ is nano semi closed set.

Proposition 2.2 Let $\mathfrak{F}$ be nano clopen subset of nano topological space $(\mathfrak{W}, \mathrm{T}_{\mathscr{b}}((\mathfrak{X})))$ $\mathfrak{V} \subseteq \mathfrak{W}$ be nano α-open set then $\mathfrak{V}$ is nano clopen set.

Proof. Evident.

3. Mightily nano simply continuous and nano simply continuous functions

Definition 3.1 A nano function $f: (U, \mathrm{T}_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, \mathrm{T}_{\mathfrak{w}}((\mathfrak{Z}))$ called nano mightily simply continuous if for each nano semi-open set $\mathfrak{W}$ in $\mathfrak{V}$ (resp., nano semi closed $\mathfrak{Q}$) of $\mathfrak{V}$, $f^{-1}(\mathfrak{W})$ (resp. $f^{-1}(\mathfrak{Q})$) is nano simply open in U.

Definition 3.2 A nano function $f: (U, \mathrm{T}_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, \mathrm{T}_{\mathfrak{w}}((\mathfrak{Z}))$ is called:

1. nano simply continuous if $f^{-1}(\mathfrak{W})$ is nano simply open in U for each nano open set $\mathfrak{W}$ of $\mathfrak{V}$.
2. nano simply irresolute if $f^{-1}(\mathfrak{W})$ is nano simply open in U for each nano simply open set $\mathfrak{W}$ of $\mathfrak{V}$.

Remark 3.1 Clarifying we can show that be the nano simply-continuity was defined in expressions of nano simply open sets.

Theorem 3.1 For a nano function $f: (U, \mathrm{T}_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, \mathrm{T}_{\mathfrak{w}}((\mathfrak{Z}))$. The following conditions are equivalent:

1. f is mightily nano simply continuous.
2. For every nano semi-closed set $\mathfrak{W}$ of $\mathfrak{V}$, $f^{-1}(\mathfrak{W})$ is nano simply open in U.

Proof. Let $1 \longrightarrow 2$ and $\mathfrak{W} \in \mathfrak{S}_n O(\mathfrak{V})$ and since f is mightily nano continuous. Then $f^{-1}(\mathfrak{W}) \in \overset{\mathfrak{N}}{SO}(\mathfrak{U})$.

$2 \longrightarrow 1$ Let $\mathfrak{E} \in \mathfrak{S}_n C(\mathfrak{V})$ since $f^{-1}(\mathfrak{E}) \in \overset{\mathfrak{N}}{SO}(\mathfrak{U})$. Therefore f is mightily nano simply continuous.

Theorem 3.2 For a nano function $f: (\mathfrak{U}, \mathrm{T}_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, \mathrm{T}_{\mathfrak{w}}((\mathfrak{Z}))$. The following conditions are maintained:

1. Each nano irresolute function is mightily nano simply continuous.
2. Each mightily nano simply continuous is nano simply continuous.

Proof.

1. Let $\mathfrak{H} \in \mathfrak{S}_n O(\mathfrak{V})$ and since f is nano irresolute, then $f^{-1}(\mathfrak{H}) \in \mathfrak{S}_n O(\mathfrak{U})$, whereas all nano semi open set is simply open set, thus f is mightily nano simply continuous $f^{-1}(\mathfrak{H}) \in \overset{\mathfrak{N}}{SO}(\mathfrak{U})$. Thus f mightily nano simply continuous.
2. Let $\mathfrak{H}$ be nano open sets in $\mathfrak{V}$, whereas all nano open set is nano semi open set since f is mightily nano simply continuous then $f^{-1}(\mathfrak{H}) \in \overset{\mathfrak{N}}{SO}(\mathfrak{U})$. Thus f is nano simply continuous.

The relationship between the new concepts and the other concepts is depicted in the figure 2.

nano irresolute $\longrightarrow$ mightily nano simply continuous $\longleftarrow$ nano simply irresolute
$\searrow$
nano continuous $\longrightarrow$ nano alpha-continuous $\longrightarrow$ nano simply continuous

Figure 2. The types of nano simply

None of the consequences in the proposition can be reversed, as demonstrated by our next two cases.

Example 3.1 Let $\mathfrak{W} = \{s, \sigma, v, \mathfrak{r}\}, \frac{\mathfrak{W}}{\mathfrak{w}} = \{\{s, \mathfrak{r}\}, \{\sigma\}, \{v\}\}$, $\mathfrak{X} = \{s, v\}$ then the nano topology on U with resect to $\mathfrak{X}$ is $T_{\mathfrak{w}}(\mathfrak{X}) = \{\mathfrak{W}, \emptyset, \{v\}, \{v, \mathfrak{r}\}, \{s, v, \mathfrak{r}\}\}$. And let $\mathfrak{V} = \{\mathfrak{v}, q, \mathfrak{g}, \mathfrak{p}\}$, $\mathfrak{Z} = \{\mathfrak{g}, \mathfrak{p}\}$ and $\frac{\mathfrak{V}}{\mathfrak{w}} = \{\{\sigma\}, \{q\}, \{\mathfrak{g}\}, \{\mathfrak{p}\}\}$, subsequently the nano topology on $\mathfrak{W}$ with respect to $\mathfrak{X}$ is $T_{\mathscr{b}}(\mathfrak{Z}) = \{\mathfrak{V}, \emptyset, \{\mathfrak{g}, \mathfrak{p}\}\}$, define a function $\omega: (\mathfrak{W}, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ such that $\omega(s) = q$, $\omega(\sigma) = \mathfrak{v}$ $\omega(v) = \mathfrak{g}$ and $\omega(\mathfrak{r}) = \mathfrak{p}$ subsequently, $\omega$is nano simply continuous but not ω is mightily nano simply-continuous since there is a $\{\mathfrak{v}, q, \mathfrak{p}\}$ is nano semi open in $\mathfrak{V}$ but $\omega^{-1}(\{\mathfrak{v}, q, \mathfrak{p}\}) = \{s, \sigma, v\} \notin \overset{\mathfrak{N}}{SO}(\mathfrak{W}\}$.

Example 3.2 Let $\mathfrak{W} = \{\mathfrak{c}, s, \mathfrak{d}, \mathfrak{s}\}$, $\frac{\mathfrak{W}}{\mathfrak{w}} = \{\{\mathfrak{c}, \mathfrak{s}\}, \{s\}, \{\mathfrak{d}\}\}$, $\mathfrak{X} = \{\mathfrak{c}, \mathfrak{d}\}$ then the nano topology on $\mathfrak{W}$ with resect to $\mathfrak{X}$ is $T_{\mathscr{b}}(\mathfrak{X}) = \{\mathfrak{W}, \emptyset, \{\mathfrak{d}\}, \{\mathfrak{d}, \mathfrak{s}\}, \{\mathfrak{c}, \mathfrak{d}, \mathfrak{s}\}\}$. And let $\mathfrak{V} = \{\mathfrak{O}, \mathfrak{D}, w, Q\}$, $\mathfrak{Z} = \{\mathfrak{O}, Q\}$ and $\frac{\mathfrak{V}}{\mathfrak{w}} = \{\{\mathfrak{O}\}, \{\mathfrak{D}\}, \{w\}, \{Q\}\}$, then the nano topology on $\mathfrak{W}$ with resect to $\mathfrak{X}$ is $T_{\mathfrak{w}}(\mathfrak{Z}) = \{\mathfrak{V}, \emptyset, \{\mathfrak{O}, Q\}\}$, define a function $\omega: (\mathfrak{W}, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ such that $\omega(\mathfrak{c}) = \mathfrak{D}$, $\omega(s) = w$, $\omega(\mathfrak{d}) = \mathfrak{O}$ and $\omega(\mathfrak{s}) = Q$ then ω is mightily nano simply continuous but not ω is nano simply irresolute since there is a $\{\mathfrak{O}, \mathfrak{D}, w\} \in \overset{\mathfrak{N}}{SO}(\mathfrak{V}\}$ but $\omega^{-1}(\{\mathfrak{O}, \mathfrak{D}, w\}) = \{\mathfrak{c}, s, \mathfrak{d}\} \notin \overset{\mathfrak{N}}{SO}(\mathfrak{W}\}$.

Example 3.3 Let $\mathfrak{V} = \{\mathfrak{O}, \mathfrak{D}, w, Q\}$, $\mathfrak{Z} = \{\mathfrak{O}, w\}$ and $\frac{\mathfrak{V}}{\mathfrak{w}} = \{\{\mathfrak{O}\}, \{\mathfrak{D}\}, \{w\}, \{Q\}\}$ then the nano topology on $\mathfrak{W}$ with resect to $\mathfrak{X}$ is $T_{\mathfrak{w}}(\mathfrak{Z}) = \{\mathfrak{V}, \emptyset, \{\mathfrak{O}, w\}\}$, and let $\mathfrak{W} = \{\mathfrak{e}, n, y, z\}$, $\frac{\mathfrak{W}}{\mathfrak{w}} = \{\{\mathfrak{e}, z\}, \{n\}, \{y\}\}$, $\mathfrak{X} = \{\mathfrak{e}, y\}$ then the nano topology on $\mathfrak{W}$ with respect to $\mathfrak{X}$ is $T_{\mathfrak{w}}(\mathfrak{X}) = \{\mathfrak{W}, \emptyset, \{\mathfrak{e}\}, \{n, y\}, \{\mathfrak{e}, n, y\}\}$ define a function $\mathcal{H}: (\mathfrak{W}, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ such that $\mathcal{H}(\mathfrak{e}) = \mathfrak{O}, \mathcal{H}(n) = \mathfrak{D}$, $\mathcal{H}(y) = w$ and $\mathcal{H}(Q) = z$, then $\mathcal{H}$is nano simply continuous but not $\mathcal{H}$ neither nano alpha-continuous since there is $\{\mathfrak{O}, w\}$ is a nano open in $\mathfrak{V}$ but $\mathcal{H}^{-1}(\{\mathfrak{O}, w\}) = \{\mathfrak{e}, z\} \notin \overset{\mathfrak{N}}{\alpha O}(\mathfrak{W}\}$ nor nano irresolute since there is $\{\mathfrak{O}, w\}$ is nano semi open set in $\mathfrak{W}$ but $\mathcal{H}^{-1}(\{\mathfrak{O}, w\}) = \{\mathfrak{e}, z\}$ is not semi open semi in $\mathfrak{W}$

Theorem 3.3 A function $f: (U, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ is nano simply irresolute if and only if it is nano simply continuous.

Proof. let $\mathfrak{H}$ be nano open set in $\mathfrak{V}$ hence $\mathfrak{H}$ is nano semi open set since f is nano irresolute then $f^{-1}(\mathfrak{E})$ is nano semi open set in $\mathfrak{U}$ ase very nano semi open set is nano simply open thus $f^{-1}(\mathfrak{E}) \in \overset{\mathfrak{N}}{SO}(\mathfrak{U})$, then f is nano simply continuous.

Theorem 3.3 If $f: (U, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ and $g: (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z})) \longrightarrow (\mathfrak{L}, T_{\mathfrak{w}}((\mathfrak{T}))$ are two nano functions then:

1. If $f: (U, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ is nano irresolute and $g: (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z})) \longrightarrow (\mathfrak{L}, T_{\mathfrak{w}}((\mathfrak{T}))$ is nano, simply continuous. Then $g \circ f$ is nano simply continuous.
2. if $g \circ f$ is nano simply continuous and g is nano simply continuous. Then f is nano open map.

Proof.

1. If $f: (U, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ is nano simply continuous and $g: (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z})) \longrightarrow (\mathfrak{L}, T_{\mathfrak{w}}((\mathfrak{T}))$ is nano continuous consequently, $g \circ f$ is nano simply continuous.

Since g is nano continuous, then for every Q is nano open set in $\mathfrak{L}$ thus $g^{-1}(Q) \in \overset{\mathfrak{N}}{SO}(\mathfrak{V})$ since f is nano simply irresolute, consequently, $f^{-1}(g^{-1}(Q)) \in \overset{\mathfrak{N}}{SO}(U)$ but $f^{-1}(g^{-1}(Q)) = (g \circ f)^{-1}(Q) \in \overset{\mathfrak{N}}{SO}(U)$. Thus f is nano simply continuous.

2. since $g \circ f$ is nano open map then for every nano open set $\mathcal{M}$ in U, consequently, $g \circ f(\mathcal{M})$ is nano open set in $\mathfrak{V}$ due to g is nano simply continuous, consequently, $g^{-1}(g \circ f(\mathcal{M})) = f(\mathcal{M})$ is nano open set in consequently, f is nano open map.

Proposition 3.1 If $\mathcal{G}: (U, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ is nano semi continuous consequently $\mathcal{G}$ is nano simply continuous.

Proof. As $\mathcal{G}$ is nano semi continuous, then for each $\mathcal{N}$ is nano open set in $\mathfrak{V}$ then $\mathcal{G}^{-1}(\mathcal{N}) \in \mathfrak{S}_n O(\mathfrak{V})$ as each nano semi open set is nano simply open thus $\mathcal{G}$ is nano simply continuous.

Theorem 3.4 If $\mathcal{G}: (U, T_{\mathfrak{w}}((\mathfrak{X})) \longrightarrow (\mathfrak{V}, T_{\mathfrak{w}}((\mathfrak{Z}))$ is nano irresolute then $\mathcal{G}$is mightily nano simply continuous.

Proof. Let $1 \longrightarrow 2$ and $\mathfrak{Y} \in \mathfrak{S}_n O(\mathfrak{V})$, then $\mathcal{G}^{-1}(\mathcal{N}) \in \mathfrak{S}_n O(U)$, whereas nano semi open set is nano simply open set thus $\mathcal{G}$ is mightily nano continuous.

4. Conclusion

Nano simply open sets were presented in this work, along with their characteristics and connections to other sets were thoroughly examined. This study looks at an area of nano topological space (NTS) that has not been looked at as much. Medical decision-making can be qualitatively analyzed using nano-open sets. This study paper focuses on, introduce a novel concept known as the nano simply alpha open set and propose a new approximation space that extends Pawlak's approximation space. This new approximation space is referred to as nano simply alpha and nano simply alpha upper. Using these sets, we defined new types of continuous functions, namely nano simply continuous, nano simply irresolute, and mightily simply continuous functions. Additionally, we analyzed their properties and explored their relationships with other existing types of continuous functions. Future research on this topic aims to extend Pawlak's approximations by introducing a new generalization of this concept. This advancement has potential applications in diverse areas, including decision-making and real-life challenges, particularly in the medical fields. The field of medical diagnosis is set to undergo a revolution with the integration of nano topology, which will result in significant improvements in patient outcomes and healthcare more generally. Ongoing research in this domain is expected to yield further advancements and benefits, which can be used for any number of patients and for any issue in life.

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